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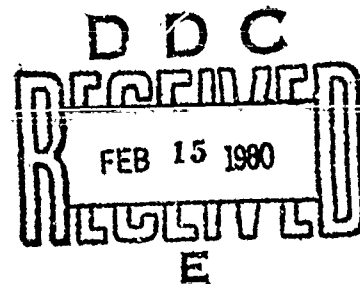


## REDUCED-ORDER OBSERVER MODEL FOR AAA TRACKER RESPONSE

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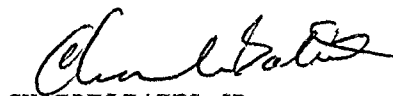
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This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

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FOR THE COMMANDER



CHARLES BATES, JR.  
Chief  
Human Engineering Division  
Air Force Aerospace Medical Research Laboratory

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## SUMMARY

A systematic study of threat effectiveness for antiaircraft artillery (AAA) systems requires the development of a mathematical model for the gunner's tracking response. The gunner (or tracker) model is then incorporated into computer simulation programs for predicting aircraft attrition with respect to specific antiaircraft weapon systems. Two of the fundamental design requirements of a gunner model are simplicity in model structure and accuracy in the tracking error predictions. A simple gunner model structure will shorten computer simulation execution time. Obviously, accurate predictions of tracking error implies model fidelity with respect to describing the gunner's tracking performance. The Luenberger full-order observer theory has been applied to design a human operator model (observer model) for AAA tracker response which has been documented in a previous report. This technical report will describe the development of an anti-aircraft gunner model based on the Luenberger reduced-order observer theory. It satisfies both the design requirements mentioned above. It is composed of three main parts - a reduced-order observer, a feedback controller, and a remnant element.

A parameter identification program based on the least squares curve-fitting method and the Gauss-Newton gradient algorithm is developed to systematically determine the model parameters. This program iteratively adjusts the parameter values to minimize the error between the model prediction of tracking error and actual human tracking data obtained from manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio. Computer simulation results of the AAA tracking task using this model are in excellent agreement with the empirical data for several aircraft flyby and maneuvering trajectories. A comparison between this model and the optimal control model by Kleinman, Baron, and Levison is also given. This model is shown to be as accurate as the optimal control model in predicting tracking errors. In addition, the computer execution time of the AAA closed loop system simulation utilizing this model is less than 15 percent of that using the optimal control model. Therefore, this gunner model can be used accurately and efficiently in the study of the AAA effectiveness and aircraft survivability.

## PREFACE

This report documents a study performed by Systems Research Laboratories, Inc. (SRL), Dayton, Ohio, for the Aerospace Medical Research Laboratory (AMRL), Manned-Systems Effectiveness Division, Manned Threat Quantification program. This work was performed under Contract F33615-76-C-5001. The Contract Monitor was Mr. Robert E. Van Patten, and the Technical Manager was Captain Jon Hull. The SRL Project Manager was Mr. Charles McKeag.

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## Section I

### INTRODUCTION

A computer simulation study of threat effectiveness for antiaircraft artillery (AAA) systems requires the development of a mathematical model for the gunner (or tracker or human operator) response. The gunner model represents the human operator's control characteristics in a compensatory tracking task. It is then incorporated into computer engagement simulation programs [1] for predicting aircraft attrition with respect to specific antiaircraft weapon systems. Two of the fundamental design requirements of a gunner model are simplicity in model structure and accuracy in the tracking error predictions. A simple gunner model structure will shorten computer simulation execution time. Obviously, accurate predictions of tracking error implies model fidelity with respect to describing the gunner's tracking performance. Then, the manned threat quantification in the threat analysis will be reliable.

An antiaircraft gunner model based on the Luenberger full-order observer theory [2] was developed and documented in a previous report [3]. In this report, the Luenberger reduced-order observer theory [4], [5] is applied to develop a tracker model for AAA compensatory tracking task. It satisfies both the design requirements mentioned above. The structure of the model is simple and its predictions of tracking errors are accurate. It is composed of three main parts - a reduced-order observer, a feedback controller, and a remnant element. An observer is itself a dynamic system whose output can be used as an estimate of the state of a given system. A reduced-order observer has dynamic order less than the observed system and provides an estimate of those state components which are not available for direct measurement. The structure of a reduced-order observer is simple and its design is easy. The idea of using a reduced-order observer in the tracker model design is to obtain an appropriate estimate of the state components (which are not directly measurable) of the gunsight system and the target motion. When the gunner (a human operator) observes the tracking error from the visual display, he not only obtains the tracking error information, but also has a certain understanding or knowledge about other

variables (state components) of the overall system. It is one of the main differences between human tracking and machine tracking. A human operator (gunner) can always realize more information about the system than what is on the display. This fact is represented by a reduced-order observer in the gunner model. The estimated state components from the reduced-order observer and the observed state components from the display are then used to implement a linear feedback controller which represents the gunner's control function in the compensatory tracking task. The effects of all the randomness sources due to human psychophysical limitations and of modelling errors are lumped into one random remnant element in this model design. Another important feature of this model is that its parameters can be determined systematically instead of by trial-and-error. A parameter identification program based on the least squares curve-fitting method [6] and the Gauss-Newton gradient algorithm [7] is developed for this purpose. This program iteratively adjusts the parameter values to minimize the least squares error between the model prediction of tracking error and actual human tracking data obtained from manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, WPAFB, Ohio. Thus it provides a convenient procedure for model validation. In addition, a computer simulation program ROOMS (Reduced-Order Observer Model Simulation) is developed with the designed model describing the gunner's response for a given AAA tracking task. The program provides time functions of the ensemble mean and standard deviation for the model's tracking error predictions (azimuth and elevation). Computer simulation results are in excellent agreement with the empirical data. Furthermore, this model is a predictive model in the sense that it can be used to predict tracking errors of an AAA system for various flyby and maneuvering trajectories with similar frequency bandwidths.

A comparison between this model and the optimal control model [8], [9], and [10] (by Kleinman, Baron, Levison) is also given. It can be shown that the model based on observer theory is as accurate as the optimal control model in predicting tracking errors. In addition, the computer execution time of the AAA closed loop system simulation utilizing this model is less than 15 percent of that using the optimal control model. This is a primary advantage of a model with simple structure.

The design of the antiaircraft gunner model based on the reduced-order observer theory is described in detail in Section II. Section III gives the model validation method and computer simulation results. The conclusion is given in Section IV.

## Section II

### REDUCED-ORDER OBSERVER MODEL

In [3], the azimuth or elevation gunsight dynamics, rate-aided control dynamics and the target motion of an antiaircraft artillery (AAA) gun system have been represented by the following state space equation.

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}u + \underline{F}\ddot{\theta}_T \quad (1)$$

where  $\underline{x}$  is the state vector with two components,

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e_T(t) \\ \dot{e}_T(t) \end{bmatrix}$$

$e_T(t)$  is the tracking error, i.e., the difference between the target position angle  $\theta_T$  and the gunsight line angle  $\theta_g$ .  $\dot{e}_T$  is the target angle rate. The scalars  $u$  and  $\ddot{\theta}_T$  in Eq (1) denote the tracker's control output and target angle acceleration respectively. The matrices  $A$ ,  $B$ , and  $F$  are

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The tracking error  $e_T$  on the visual display is observed by the gunner and is expressed in the measurement equation:

$$y = C\underline{x} \quad (2)$$

where  $y$  is the observed tracking error and  $C$  is a row vector  $[1 \ 0]$ . Next, Equations (1) and (2) will be used to develop the reduced-order observer model.

The structure of the reduced-order observer model is shown in Figure 1. It consists of three main elements: a reduced-order observer, a controller, and a remnant element. The reduced-order observer processes the tracker's observation from the visual display to provide an estimate of those state components of the AAA system which are not directly measurable. It will be shown that the system equation (1) is a second order system, but the reduced-order observer is only a first order system, since some components of the state vector as given by the system outputs are already available by direct measurement. The estimation of these measurable state components is not necessary and will cause a certain degree of redundancy. The use of a reduced-order observer eliminates this redundancy and provides an approximate estimation of the state components which can't be measured directly. The controller represents the gunner's tracking function by a state variable linear feedback control law. The observer and the controller consists of the deterministic part of the gunner model. The effects of the various randomness sources in the AAA man-machine closed loop system and of the modelling errors are lumped into one element called remnant, which is the stochastic part of the gunner model. These randomness sources include the modelling error, the observation error, the neuromotor noise, etc. Mathematical equations of this model are given below.

#### A. Reduced-Order Observer Design

System equations (1) and (2) are used in the design of the first element (reduced-order observer) of the gunner model. However, the gunner does not have precise information about the target dynamics, so the term representing target acceleration,  $\ddot{\theta}_T$ , in Equation (1) will not be included in the design of the observer equation. The effect on the tracking error due to eliminating the  $\ddot{\theta}_T$  term will be included in the remnant element. Now from Equation (2),  $y = C\underline{x} = x_1$ , the tracking error is available from direct observation. Thus, it is only necessary to estimate the second component  $x_2$  of the state vector  $\underline{x}$  in order to implement a state variable feedback

# AAA GUNNER MODEL

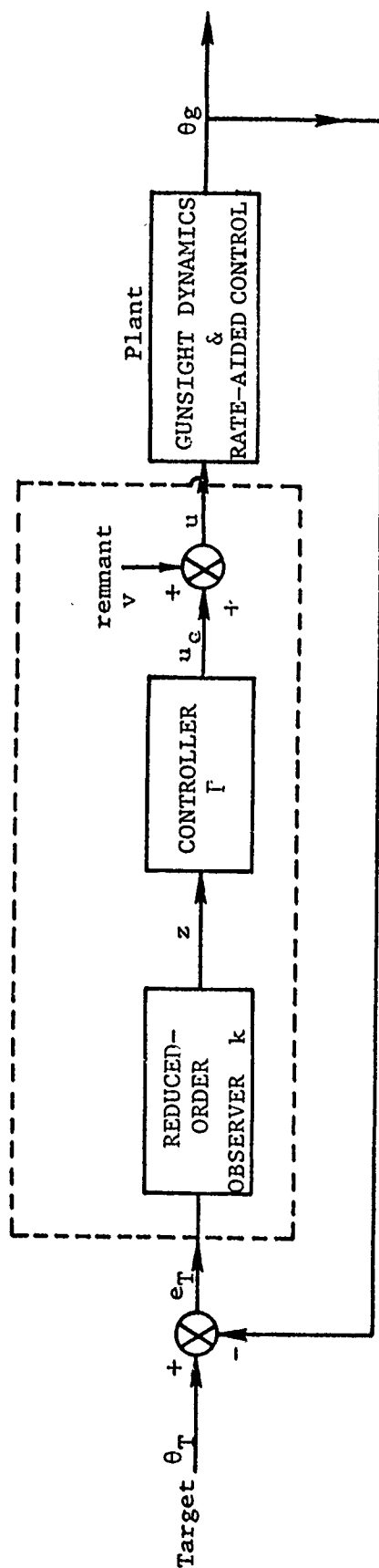


Figure 1. Block Diagram of the Structure of the Reduced-Order Observer Model

control law. In the following, the Luenberger reduced-order observer theory [4] and [5] is used to design the gunner model. First, with the  $\ddot{\theta}_T$  term eliminated, Equation (1) can be rewritten as

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1u \quad (3)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2u \quad (4)$$

Since the first state component  $x_1$  is measurable, i.e.,  $y = x_1$ , Equation (3) can also be expressed by

$$\dot{y} = a_{11}y + a_{12}x_2 + b_1u$$

or equivalently,

$$\dot{y} - a_{11}y = a_{12}x_2 + b_1u \quad (5)$$

Let us introduce a new variable  $y' = \dot{y} - a_{11}y$ , then Equations (4) and (5) can be expressed by

$$\dot{x}_2 = a_{22}x_2 + a_{21}y + b_2u \quad (6)$$

and

$$y' = a_{12}x_2 + b_1u \quad (7)$$

Now Equation (6) is the reduced-order system dynamic equation with measurement data obtained by Equation (7). Note that Equations (6) and (7) are a first order system with one measurement equation. An observer which gives an estimate  $\hat{x}_2$  of  $x_2$  can be easily designed as shown by the following equation.

$$\begin{aligned} \dot{\hat{x}}_2 &= a_{22}\hat{x}_2 + a_{21}y + b_2u_c + k(y' - a_{12}\hat{x}_2 - b_1u_c) \\ &= (a_{22} - ka_{12})\hat{x}_2 + k\dot{y} + (a_{21} - ka_{11})y + (b_2 - kb_1)u_c \end{aligned} \quad (8)$$

where  $a_{ij}$  and  $b_k$  are the elements of matrices A and B in Equation (1), the scalar  $k$  is the observer gain,  $y$  and  $\dot{y}$  are the observed tracking error and error rate respectively, and  $u_c$  is the linear feedback control law (the controller) with the form:

$$u_c = - \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$

where the feedback control gains  $\gamma_1$  and  $\gamma_2$  are two constants to be determined later. Note that the state feedback is composed of  $y$  (the observed variable which is  $x_1$ ) and  $\hat{x}_2$  (the estimated state of  $x_2$ ). It can be shown that the system (1) and (2) is completely observable. (The definition of observability and the conditions of a system to be observable can be found in [11]. Then, by the observer theory, there always exists an observer gain  $k$  to make the eigenvalue of the observer (Equation (8)) negative. Thus, the output of the observer will be a good estimation to the state of the observed system. This shows the existence of proper observer gain  $k$  in Equation (8). Actually, the value of observer gain  $k$  is determined by a curve-fitting identification program. The required differentiation of  $y$  in Equation (8) can be avoided by introducing the following variable:

$$z(t) = \hat{x}_2 - ky(t) \tag{9}$$

Hence the observer dynamics can be represented by

$$\begin{aligned} \dot{z} = & (a_{22} - ka_{12}) z + (a_{22} - ka_{12}) ky + (a_{21} - ka_{11}) y + \\ & (b_2 - kb_1) u_c \end{aligned} \tag{10}$$

Next, the actual output of this model is expressed as the sum of the output  $u_c$  of the controller and the remnant element  $v$ .

$$\begin{aligned} u = & u_c + v \\ = & - \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix} + v \end{aligned} \tag{11}$$

where the remnant term  $v(t)$  is modeled as a white noise and its statistical properties are selected to be

$$E [v(t)] = 0 \quad \text{for all } t$$

$$E [v(t) v(\tau)] = q(t) \delta(t - \tau) \quad \text{for all } t \text{ and } \tau \quad (12)$$

where  $E$  is the expectation operator and the covariance function  $q(t)$  is a scalar function of time and will be described later.  $\delta$  is the Dirac delta function.

The gunner model equations of the observer, the controller, and the remnant have been derived. These equations are combined with system equations (1) and (2) to obtain the mathematical model of the closed loop AAA system. Since  $x_1 = y$ , Equations (1) and (10) can be rewritten as follows

$$\dot{y} = a_{11}y + a_{12}x_2 + b_1u + f_1\ddot{\theta}_T \quad (13)$$

$$\dot{x}_2 = a_{21}y + a_{22}x_2 + b_2u + f_2\ddot{\theta}_T$$

$$\dot{z} = (a_{22} - ka_{12}) z + (a_{22} - ka_{12}) ky + (a_{21} - ka_{11}) y +$$

$$(b_2 - kb_1) u_c$$

$$u = u_c + v$$

$$u_c = - \begin{bmatrix} \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} y \\ \hat{x}_2 \end{bmatrix}$$

By introducing new variables:

$$x_3 = x_2 - ky$$

and

$$e = x_3 - z \quad (14)$$

Equation (13) can be rewritten as

$$\begin{aligned} \dot{y} &= \left[ a_{11} + a_{12}k - b_1(\gamma_1 + k\gamma_2) \right] y + (a_{12} - b_1\gamma_2)x_3 + \\ &\quad b_1\gamma_2e + f_1\ddot{\theta}_T + b_1v \\ \dot{x}_3 &= \left[ (a_{22} - ka_{12})k + (a_{21} - ka_{11}) - (b_2 - kb_1)(\gamma_1 + k\gamma_2) \right] y + \\ &\quad \left[ a_{22} - ka_{12} - (b_2 - kb_1)\gamma_2 \right] x_3 + (b_2 - kb_1)\gamma_2e + \\ &\quad (b_2 - kb_1)v + (f_2 - kf_1)\ddot{\theta}_T \\ e &= (a_{22} - ka_{12})e + (f_2 - kf_1)\ddot{\theta}_T + (b_2 - kb_1)v \end{aligned}$$

Or equivalently,

$$\dot{X} = A_1X + F_1\ddot{\theta}_T + D_1v \quad (15)$$

where  $X$  is the state vector of the overall system with components:

$$X = \begin{bmatrix} y \\ x_3 \\ e \end{bmatrix} = \begin{bmatrix} y \\ x_2 - ky \\ x_3 - z \end{bmatrix}$$

$A_1$ ,  $F_1$ , and  $D_1$  are matrices defined as follows:

$$A_1 = \begin{bmatrix} a_{11} + a_{12}k - b_1 (\gamma_1 + k \gamma_2) & a_{12} - b_1 \gamma_2 & b_1 \gamma_2 \\ (a_{22} - ka_{12})k + a_{21} - ka_{11} - & a_{22} - ka_{12} - & (b_2 - kb_1)\gamma_2 \\ (b_2 - kb_1)(\gamma_1 + k\gamma_2) & (b_2 - kb_1)\gamma_2 & \\ 0 & 0 & a_{22} - ka_{12} \end{bmatrix}$$

$$F_1 = \begin{bmatrix} f_1 \\ f_2 - kf_1 \\ f_2 - kf_1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} b_1 \\ b_2 - kb_1 \\ b_2 - kb_1 \end{bmatrix}$$

In order to determine the parameters  $k$ ,  $\gamma_1$ ,  $\gamma_2$ , and the covariance function  $q(t)$  in Equation (12), we use the following equations. Letting the expectation value of  $X$  be  $\bar{X}$  then we have,

$$\dot{\bar{X}} = A_1 \bar{X} + F_1 \ddot{\theta}_T \quad (16)$$

and the covariance matrix of  $X(t)$  is  $P(t) = E \left[ (X(t) - \bar{X}(t)) (X(t) - \bar{X}(t))^T \right]$ ; then it can be shown in [ 11 ] that the covariance matrix is governed by

$$\dot{P} = A_1 P + P A_1^T + D_1 q(t) D_1^T \quad (17)$$

Equations (16) and (17) will be used in Section III to determine the values of parameters in the gunner model.

## B. Frequency Domain Analysis

A detailed study of the frequency domain responses of some key time domain variables of the AAA tracking system has been done. This will provide us information to design the covariance function  $q(t)$  of the remnant element (see Equation (12)). The power spectral density functions (PSD) of the target angle  $\theta_T$ , angle rate  $\dot{\theta}_T$  and angle acceleration  $\ddot{\theta}_T$  were generated for the four flyby and maneuvering trajectories [3]. These PSD functions were generated for both azimuth and elevation components of target motion variables. It was found that the significant parts of all these PSD functions are less than 0.5 Hz. In addition, similar PSD functions were generated for the sample ensemble mean  $\bar{e}_T$  of tracking errors which were obtained by averaging empirical data of sixteen runs. The empirical data was generated from manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, WPAFB. There is a significant consistency between the PSD's of target angle acceleration  $\ddot{\theta}_T$  and the PSD's of the corresponding sample ensemble mean  $\bar{e}_T$  of the tracking error. These results are shown in Figures 2 through 9. There are two curves on each of these figures. The solid curve denotes the PSD function of the empirical mean of tracking error. The dashed curve describes the corresponding PSD function of the target angle acceleration. Figures 2 through 5 show the comparison of PSD functions of the azimuth case for four target trajectories, respectively. Figures 6 through 9 show the similar results for the elevation case. The low frequency parts of the PSD functions of target accelerations match well the low frequency parts of the PSD functions of empirical means of corresponding tracking errors. The high frequency parts of the PSD functions of empirical mean tracking errors in these figures are the results due to small sample size of experimental runs. Figure 8 shows that these two PSD functions do not match. After further investigation, it was found that the data tape containing the information of target elevation acceleration was not properly generated. In this frequency domain analysis, it is concluded that the tracking error in AAA tracking task depends on the target acceleration. In other words, when the target aircraft makes a maneuvering flight (i.e., the corresponding target acceleration increases), the tracking difficulty increases and hence the tracking error increases. This observation will be used in the next section to design the covariance function of remnant element.

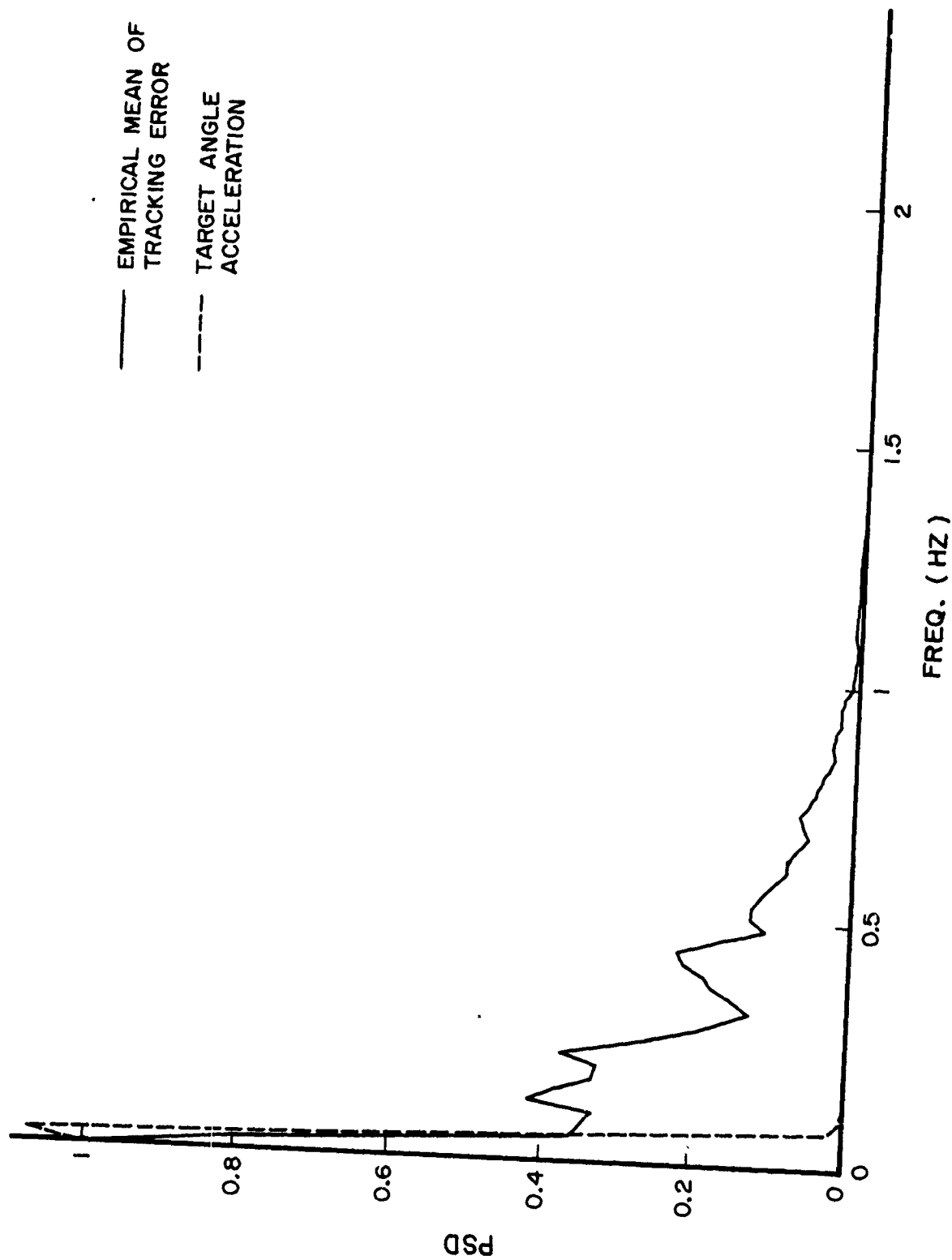


Figure 2. Comparison of PSD Functions Trajectory 1 (Azimuth)

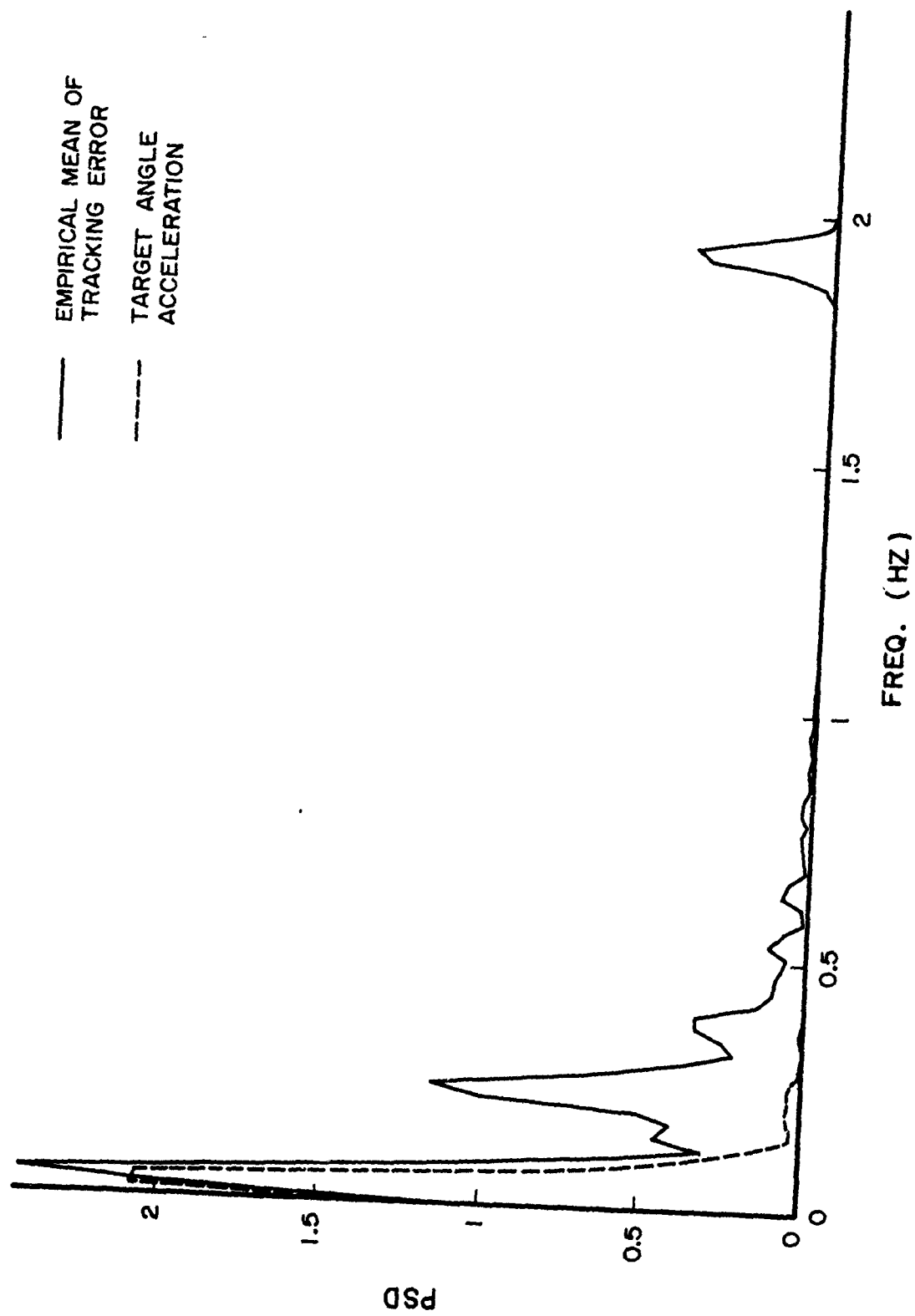


Figure 3. Comparison of PSD Functions Trajectory 2 (Azimuth)

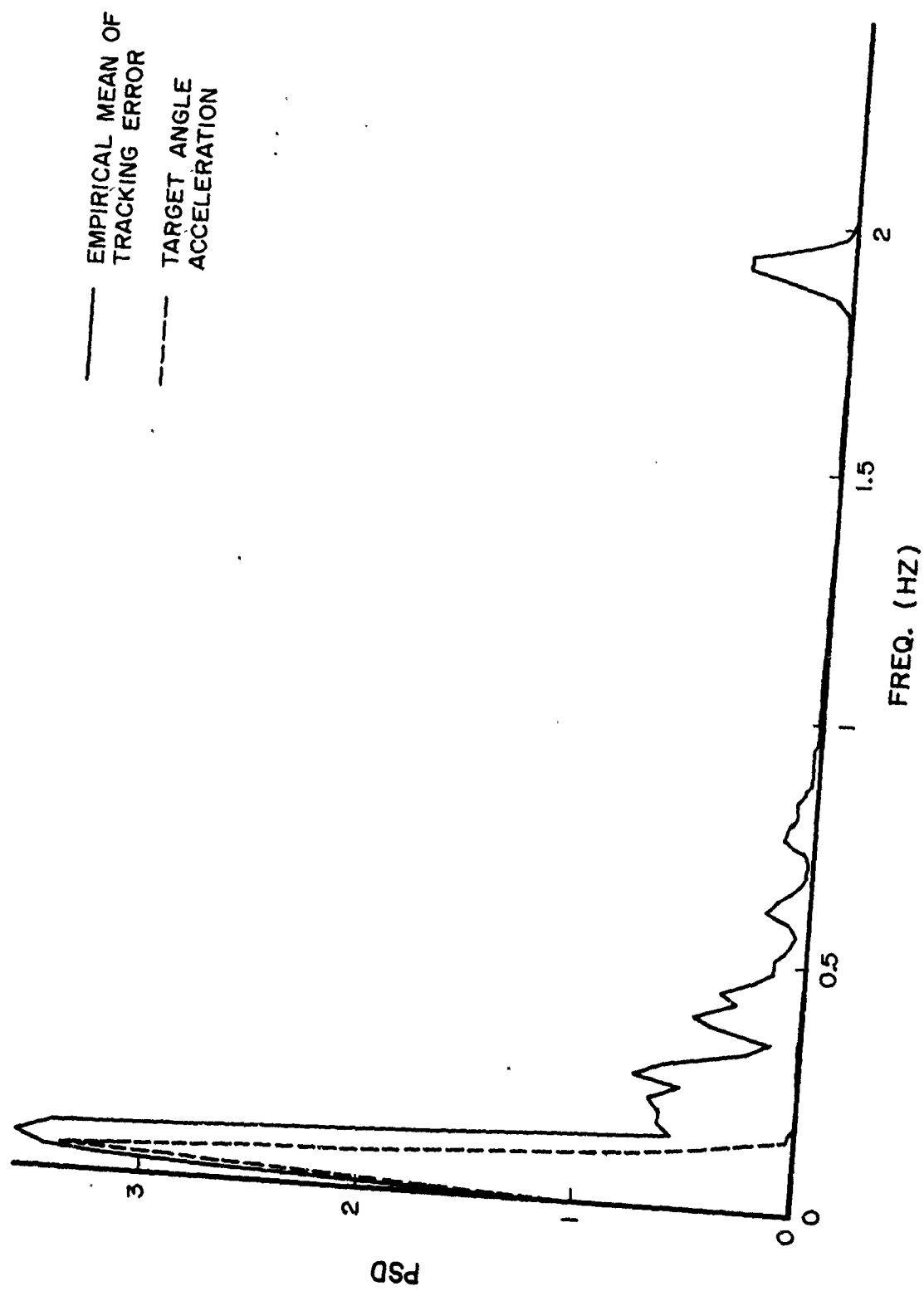


Figure 4. Comparison of PSD Functions Trajectory 3 (Azimuth)

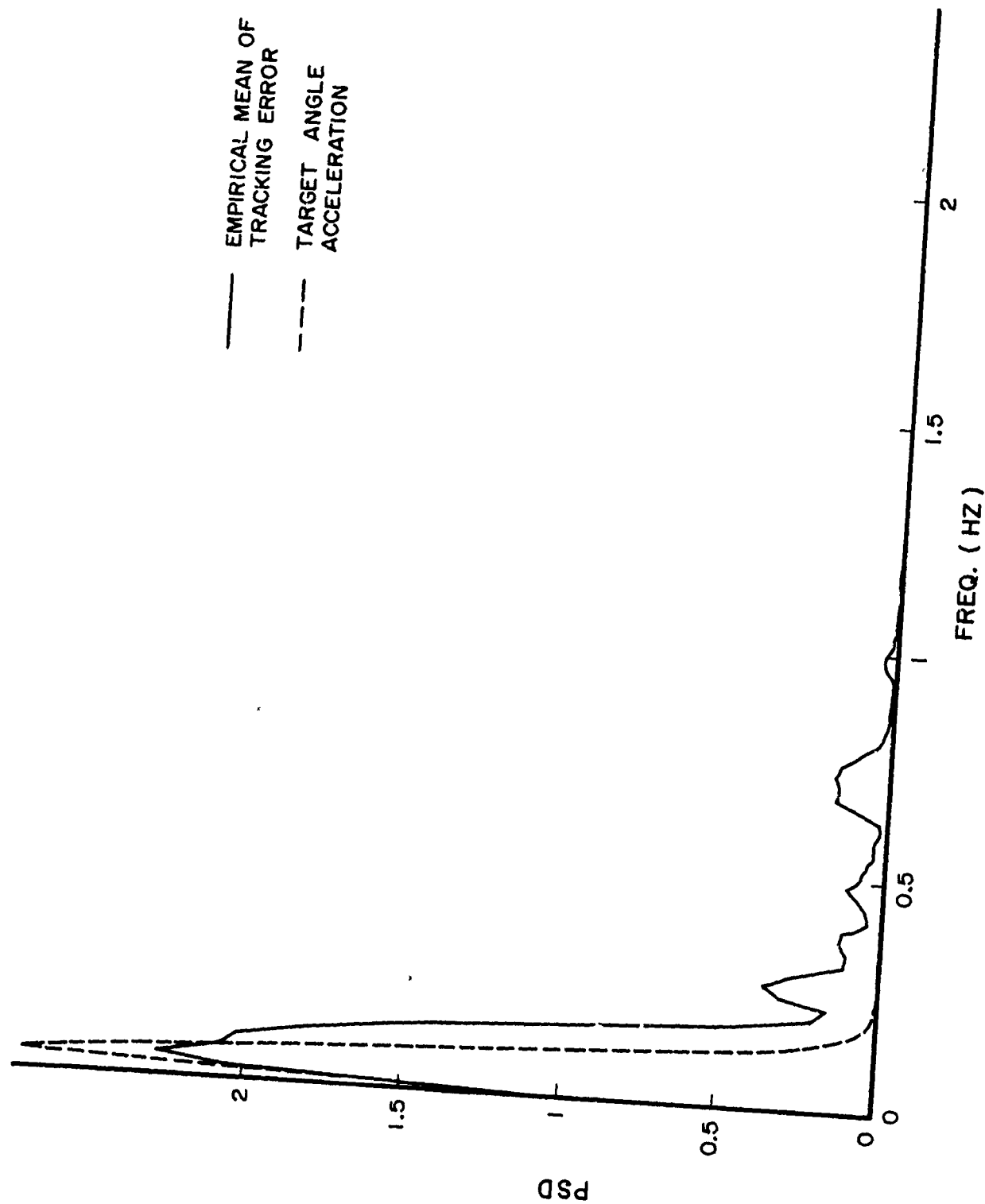


Figure 5. Comparison of PSD Functions Trajectory 4 (Azimuth)

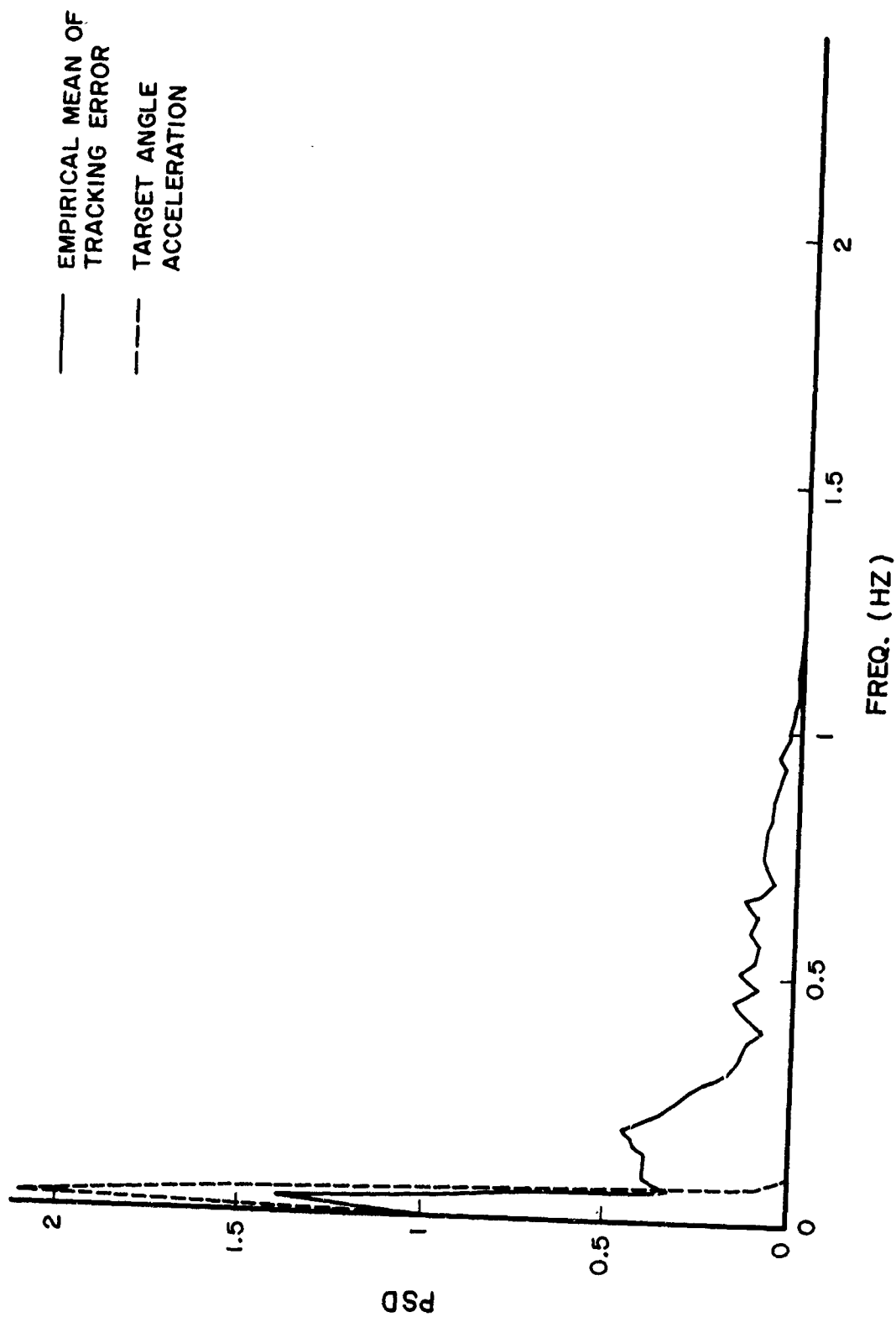


Figure 6. Comparison of PSD Functions Trajectory 1 (Elevation)

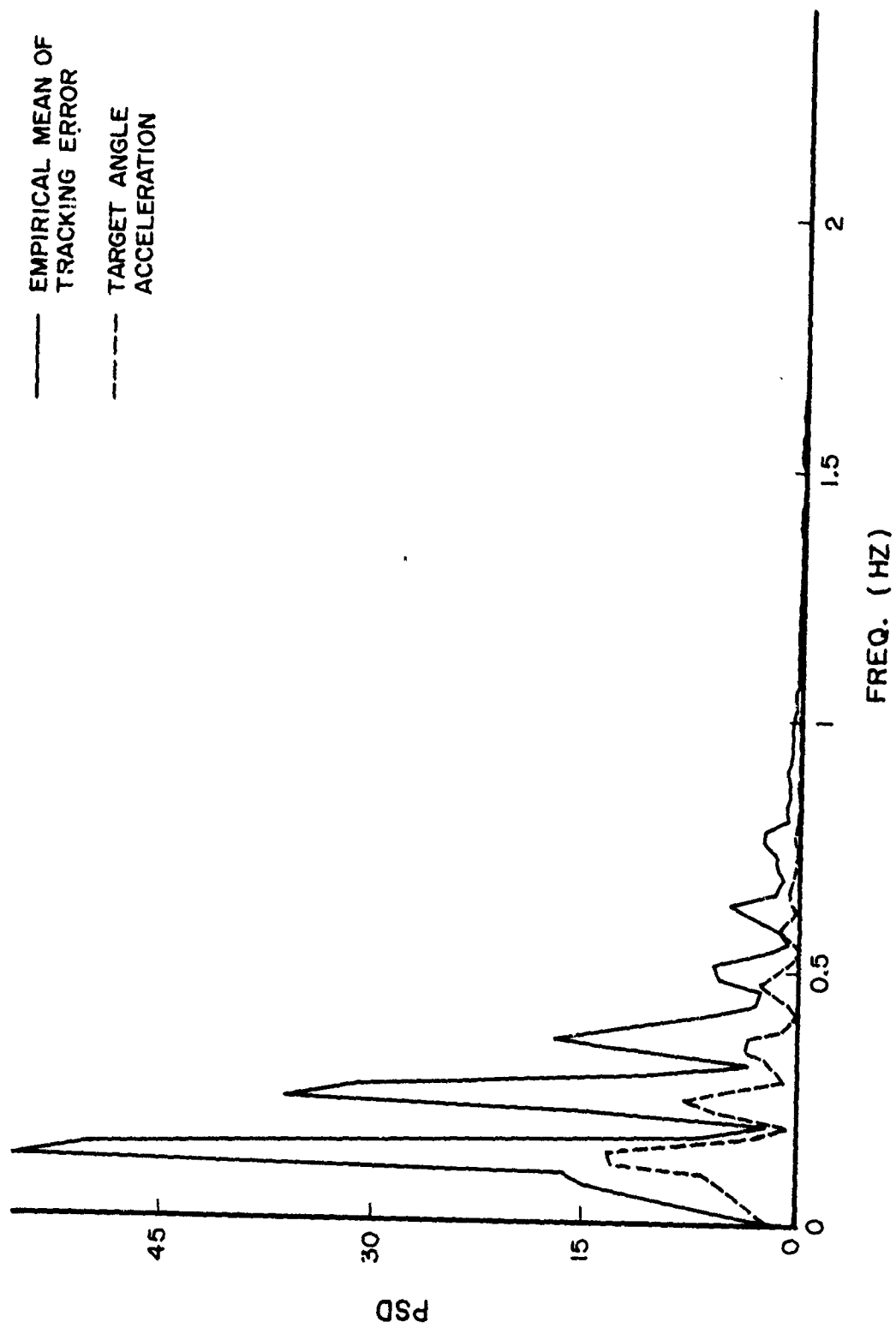


Figure 7. Comparison of PSD Functions Trajectory 2 (Elevation)

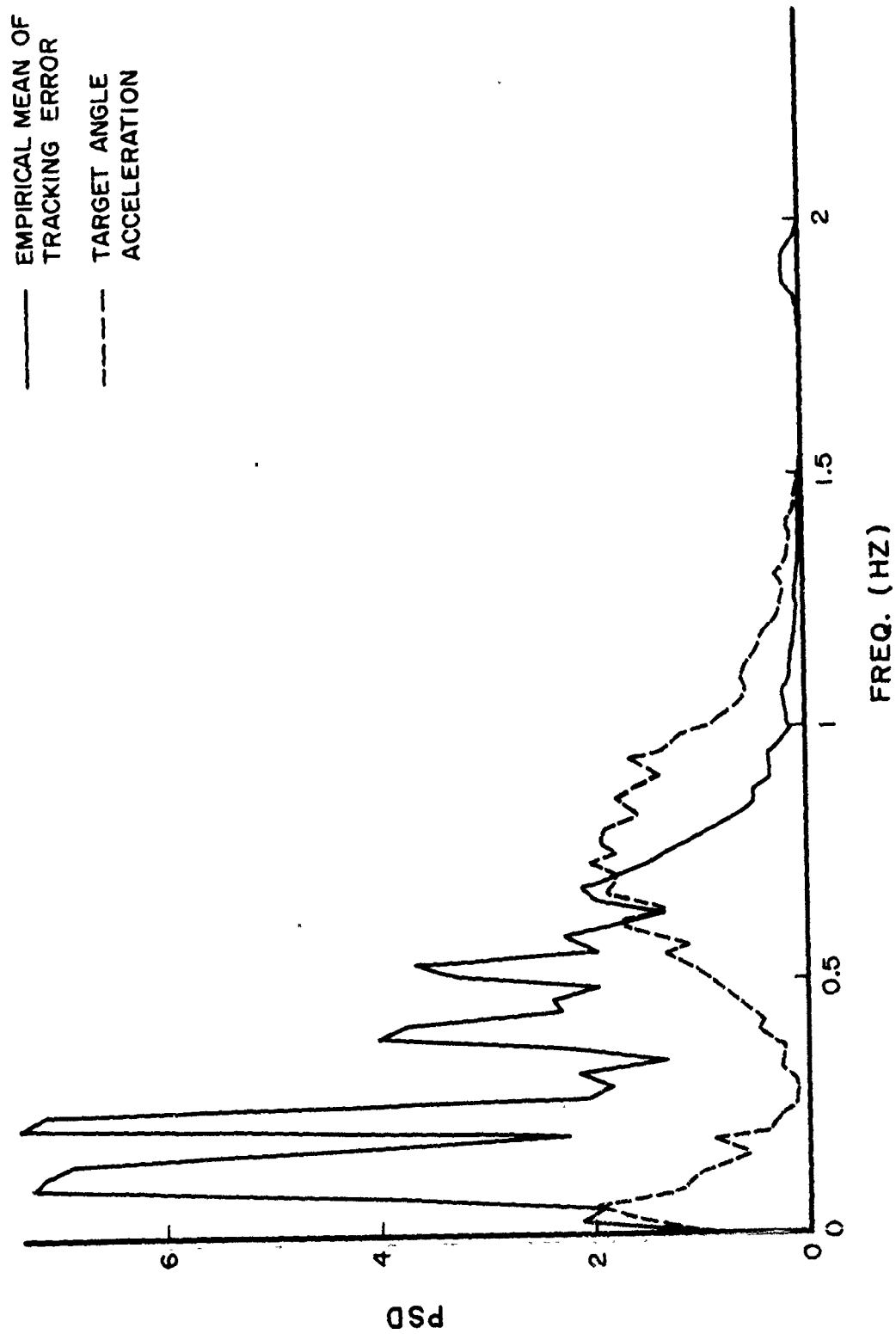


Figure 8. Comparison of PSD Functions Trajectory 3 (Elevation)

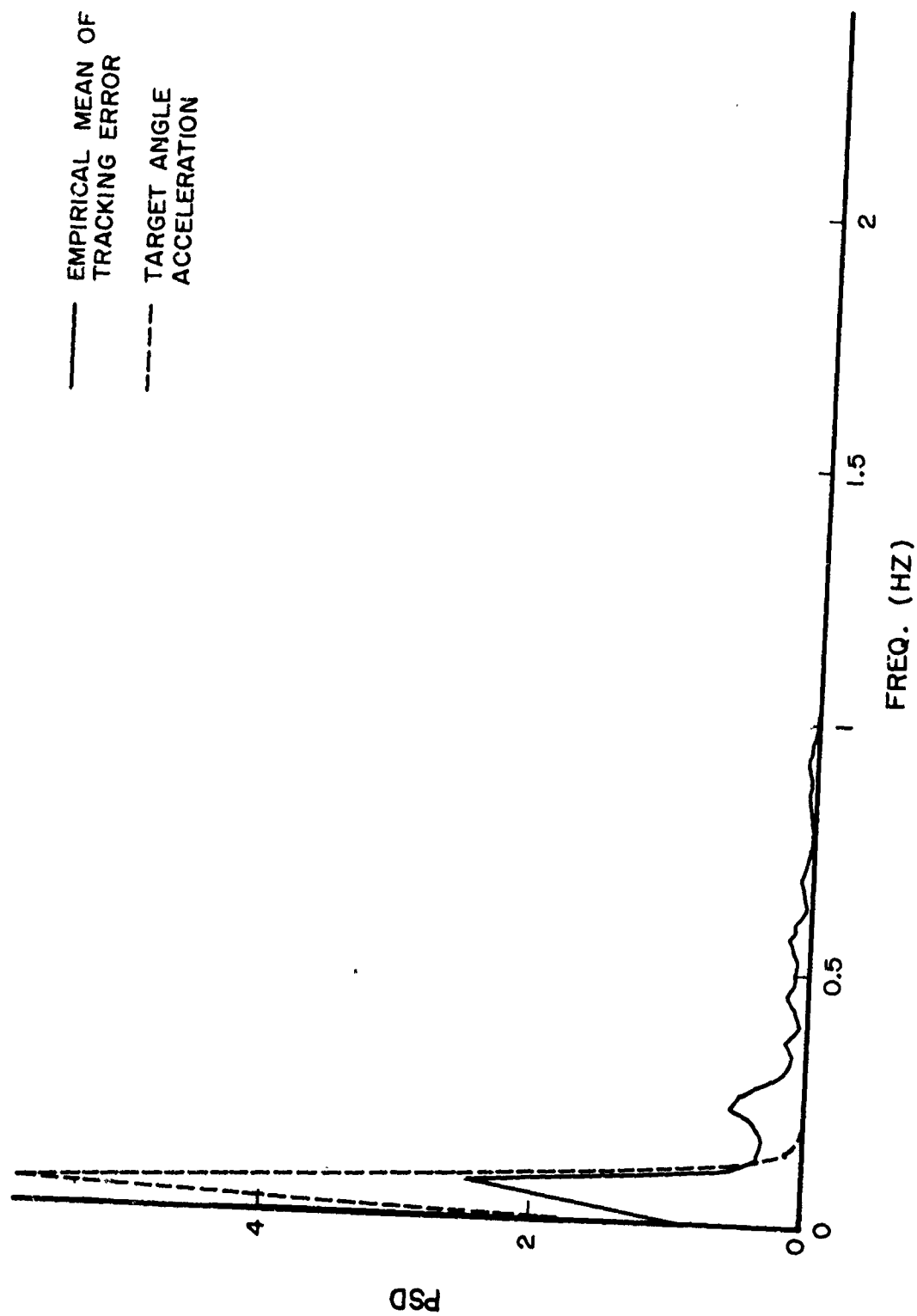


Figure 9. Comparison of PSD Functions Trajectory 4 (Elevation)

### Section III

#### MODEL VALIDATION

##### A. Time Domain Curve-Fitting Identification Method

The design of a gunner model included two main procedures; the design of the structure of the model and the determination of the values of the model parameters. The structure of the gunner model based on observer theory has been described in Section II. In this section, the parameters of the model will be determined through an identification program. Figure 10 shows the parameter determination procedure. The manned AAA simulator built at the Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio, provides the empirical tracking error data,  $e_T(t)$ , along with the simulated target trajectory  $\theta_T$  as the input.  $\bar{e}_T(t)$  represents the sample ensemble mean of the tracking error of sixteen simulation runs. Now in Equation (16), the first component of  $\bar{X}$  is the expectation value of the tracking error, i.e., the model prediction  $\bar{e}_T'$  of the ensemble mean of the tracking error in the AAA closed loop system.  $\Delta\bar{e}_T$  denotes the difference between the empirical data  $\bar{e}_T$  and the model prediction  $\bar{e}_T'$ . The criterion function evaluates the "goodness of fit" between the model prediction and the empirical data. In this study, the criterion function is selected to be

$$J = \int_0^{t_f} (\bar{e}_T(t) - \bar{e}_T'(t, \gamma_1, \gamma_2, k))^2 dt$$

where  $t_f$  is the tracking duration (equal to 45 seconds),  $\bar{e}_T'$  is the model prediction of the tracking error which is a function of time and the unknown parameters. The explicit form of the function  $\bar{e}_T'$  in terms of time  $t$  and model parameters  $\gamma_1, \gamma_2$  and  $k$  has been derived and is included in Appendix A. The criterion function  $J$  will be minimized with respect to these unknown parameters. The Gauss Newton gradient algorithm was used as the parameter adjustment algorithm to iteratively determine the values of model parameters. The values of parameters obtained through this identification program are:

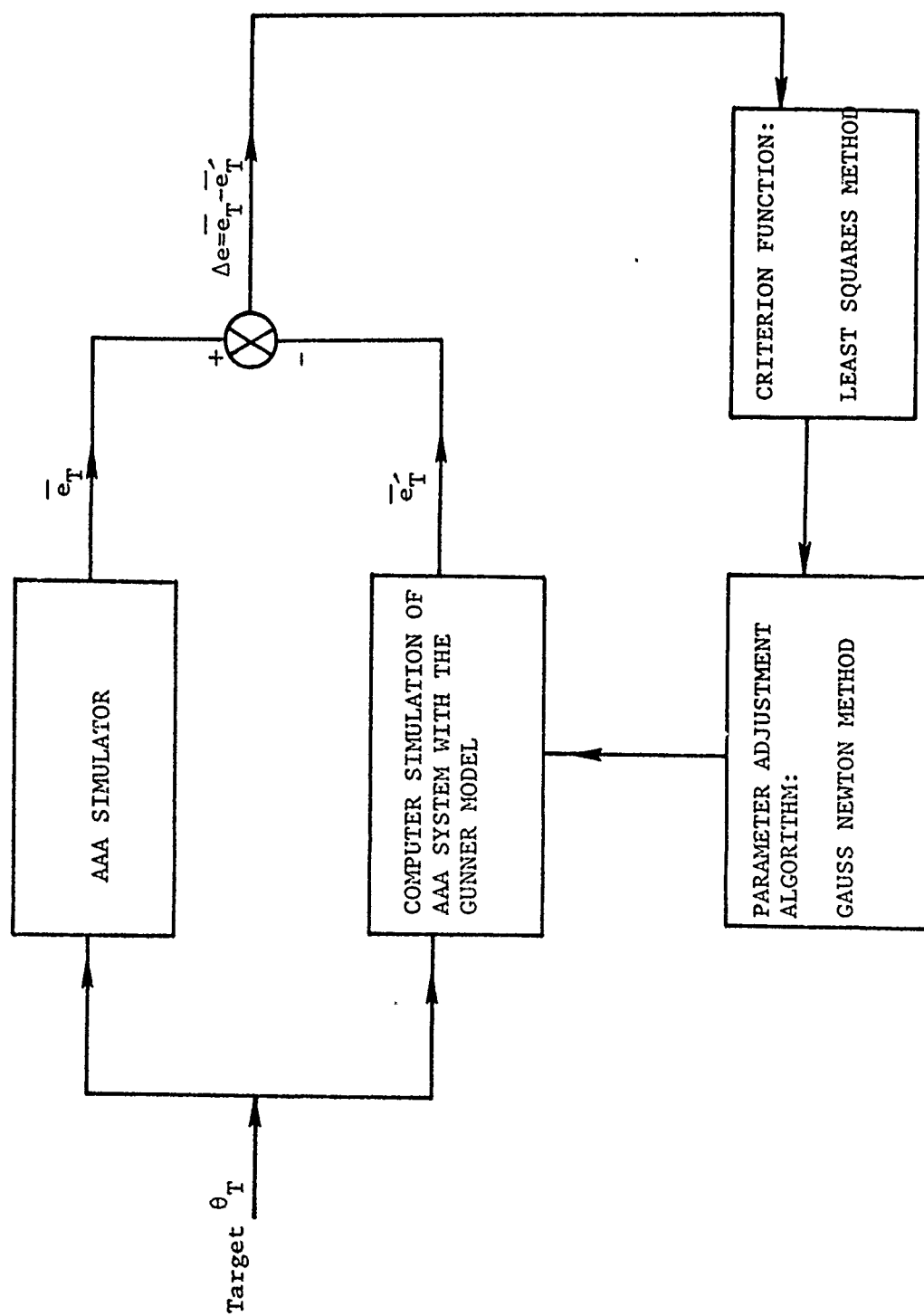


Figure 10. Block Diagram of the Parameter Identification Procedure

	Azimuth Tracking	Elevation Tracking
Observer gain k	2.94	3.02
Controller gains $\gamma_1$	-2.87	-3.01
$\gamma_2$	-1.00	-1.00

Once the observer gain and the controller gains are determined, the system matrix  $A_1$  is known. Next the model prediction of the standard deviation of the tracking error is considered. It can be shown that the square root of the first diagonal element  $p_{11}$  of the covariance matrix  $P(t)$  in Equation (17) is the standard deviation of the tracking error. The solution of Equation (17) is

$$P(t) = \phi(t, t_0) P(t_0) \phi^T(t, t_0) + \int_{t_0}^t \phi(t, \tau) D_1 q(\tau) D_1^T \phi^T(t, \tau) d\tau \quad (18)$$

where  $\phi(t, t_0)$  is the state transition matrix defined as

$$\phi(t, t_0) = e^{A_1(t-t_0)}$$

and  $q(t)$  is the covariance matrix of the random remnant. It has been found from the frequency domain analysis results that the tracking error depends on the target trajectory dynamics, especially the target acceleration  $\ddot{\theta}_T$ . Therefore, it is assumed that the covariance function  $q(t)$  of the remnant is a function of the target dynamics as follows:

$$q(t) = \alpha_1 + \alpha_2 \hat{\ddot{\theta}}_T^2(t) + \alpha_3 \hat{\ddot{\theta}}_T^2(t) \quad (19)$$

where  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are three nonnegative constants to be determined later

and  $\hat{\dot{\theta}}_T$  and  $\hat{\ddot{\theta}}_T$  are estimated target angle rate and acceleration, respectively. The reason that only estimated values  $\hat{\dot{\theta}}_T$  and  $\hat{\ddot{\theta}}_T$  were used in Equation (19) is that the gunner does not have precise information about  $\dot{\theta}_T$  and  $\ddot{\theta}_T$  (i.e., target uncertainty). These estimated quantities can be obtained as follows: Equation (9) can be rewritten by

$$\hat{\dot{\theta}}_T = z(t) + ky(t)$$

where  $z(t)$  is the output of the reduced-order observer,  $k$  is the observer gain, and  $y(t)$  is the observed tracking error. Since all these quantities are known,  $\hat{\dot{\theta}}_T$  can be computed. Next, by the first-order approximation,  $\hat{\ddot{\theta}}_T$  can be found from,

$$\hat{\ddot{\theta}}_T(t_k) = \frac{\hat{\dot{\theta}}_T(t_k) - \hat{\dot{\theta}}_T(t_{k-1})}{\Delta t}$$

where  $t_k = k\Delta t$ , and  $\Delta t = 0.04$  sec is the sampling period. Then by minimizing the following cost function  $J'$ , the parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  can be determined.

$$J' = \int_0^{t_f} [S_1(t) - S_2(t, \alpha_1, \alpha_2, \alpha_3)]^2 dt$$

where  $S_1$  is the sample ensemble standard deviation of empirical tracking error of sixteen runs and  $S_2$  is the model prediction of the ensemble standard deviation (an explicit function of time and the parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ ). Note that the curve-fitting of the standard deviation of tracking error is also done in the time domain instead of in the frequency domain as in [3]. The derivation of the function  $S_2$  from Equation (18) is included in Appendix B. The results of the parameter determination program in minimizing  $J'$  are,

	Azimuth	Elevation
$\alpha_1$	.0496	.0032
$\alpha_2$	.0024	.00047
$\alpha_3$	.103	.259

#### B. Computer Simulation Results

The numerical values of the parameters of this gunner model were determined in the previous subsection with respect to the gunsight dynamic system (Equation (1)) and a deterministic target trajectory. The gunner model is now ready to be used for computer simulation. A computer program called ROOMS simulates the AAA system with this model representing the gunner response. The input to this program is the target motion trajectory. The outputs are the model predictions of the ensemble mean and standard deviation of the tracking error. These results are plotted in Figures 11 through 26. There are two curves in each of these Figures. The solid curve denotes the empirical tracking data. The corresponding model prediction is denoted by the dotted curve.

Figures 11 through 14 give comparisons between the empirical data and the model prediction of the ensemble mean of azimuth tracking errors for four target trajectories. Obviously, the model predictions match the empirical data very well for both flyby and maneuvering trajectories. It should be emphasized that these results are obtained by using the designed gunner model with the same set of model parameters (determined in the previous subsection) applied to various trajectories. In other words, it verifies that with the same set of parameters, the gunner model based on the reduced-order observer theory can give accurate predictions of tracking errors in AAA weapon systems for various target trajectories with similar frequency bandwidths. Therefore, it is a predictive model. Next, Figures 15 through 18 give comparisons between the empirical data and the model prediction of

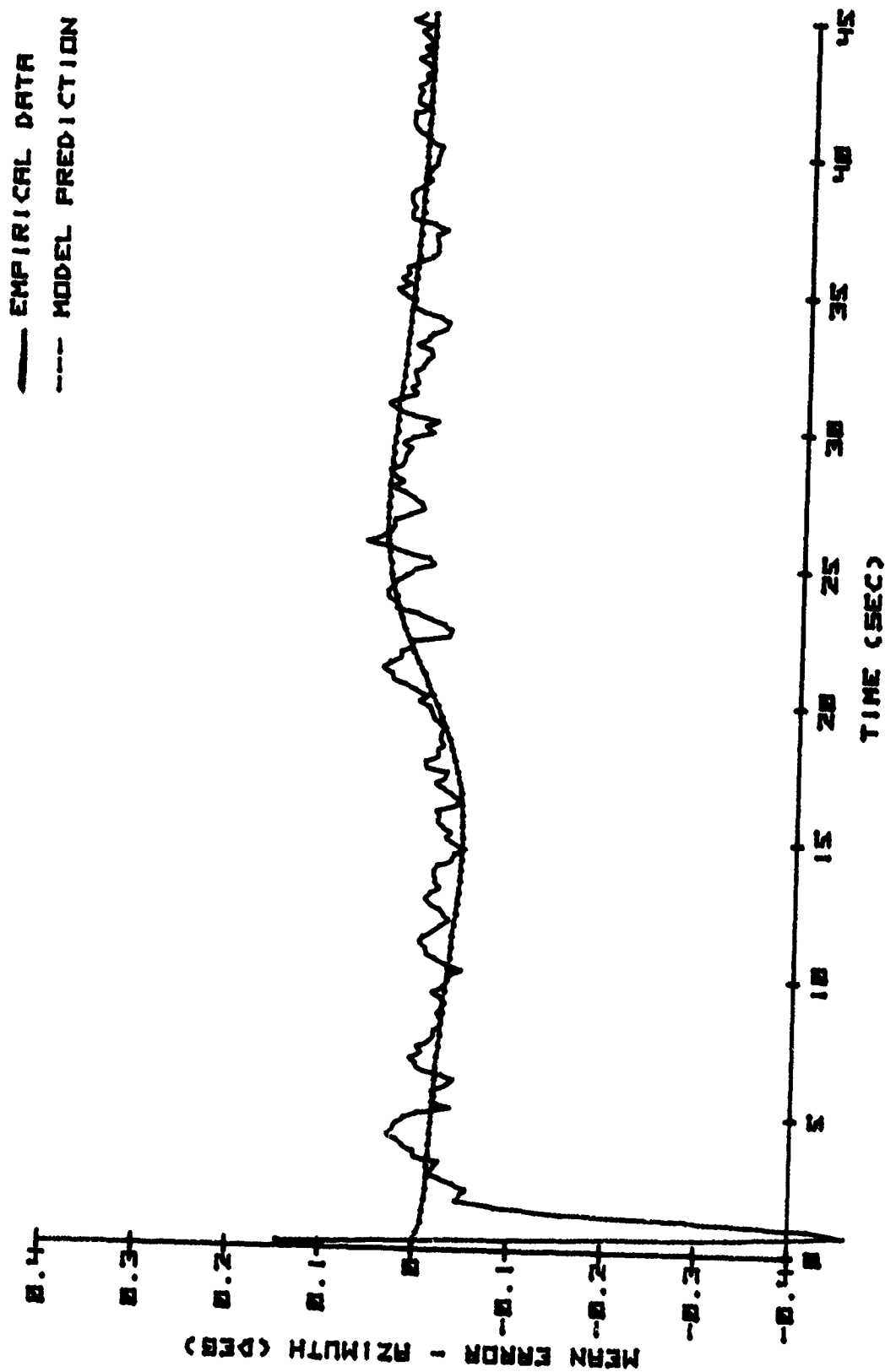


Figure 11. Mean Tracking Error Trajectory 1

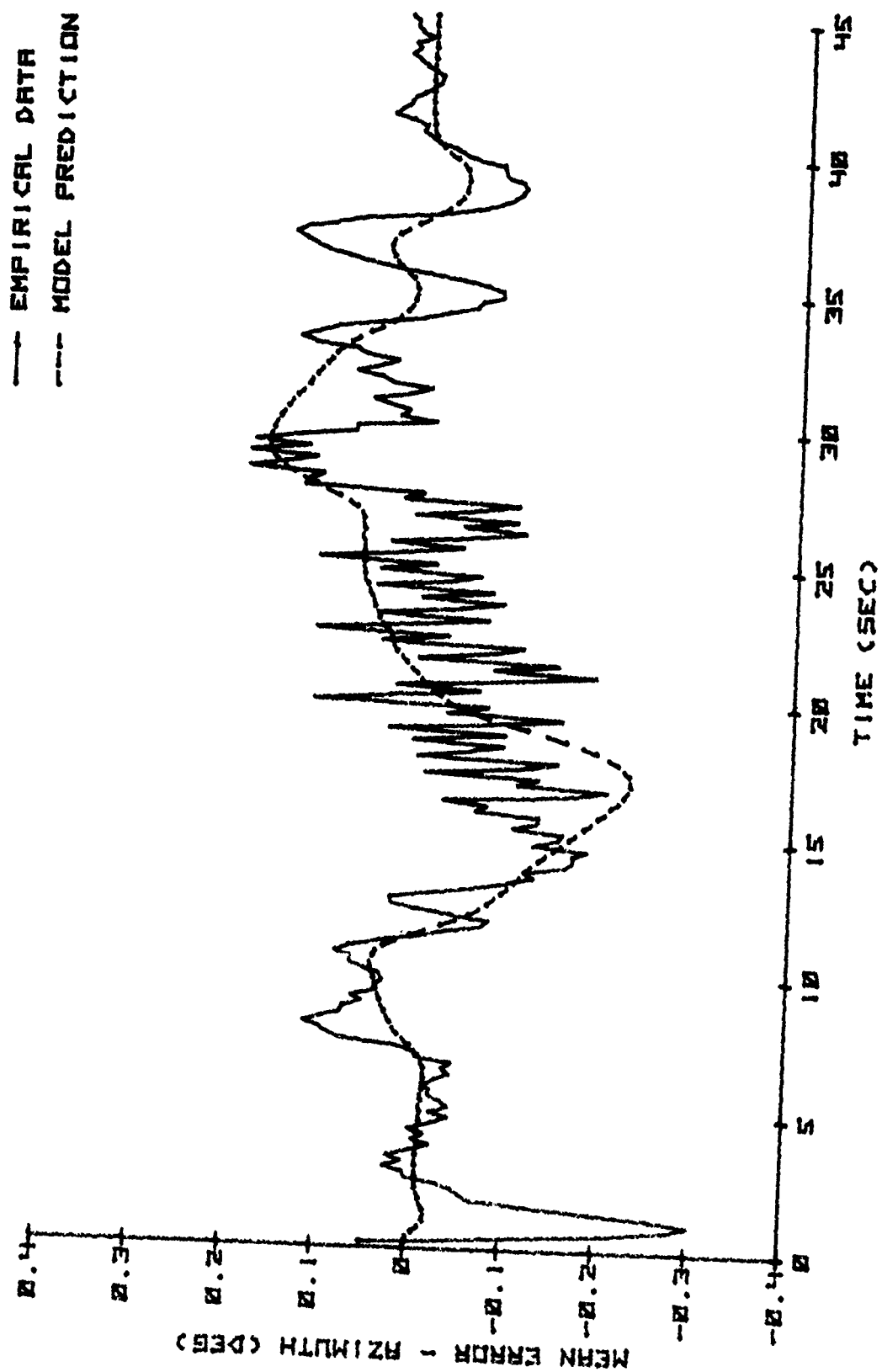


Figure 12. Mean Tracking Error Trajectory 2

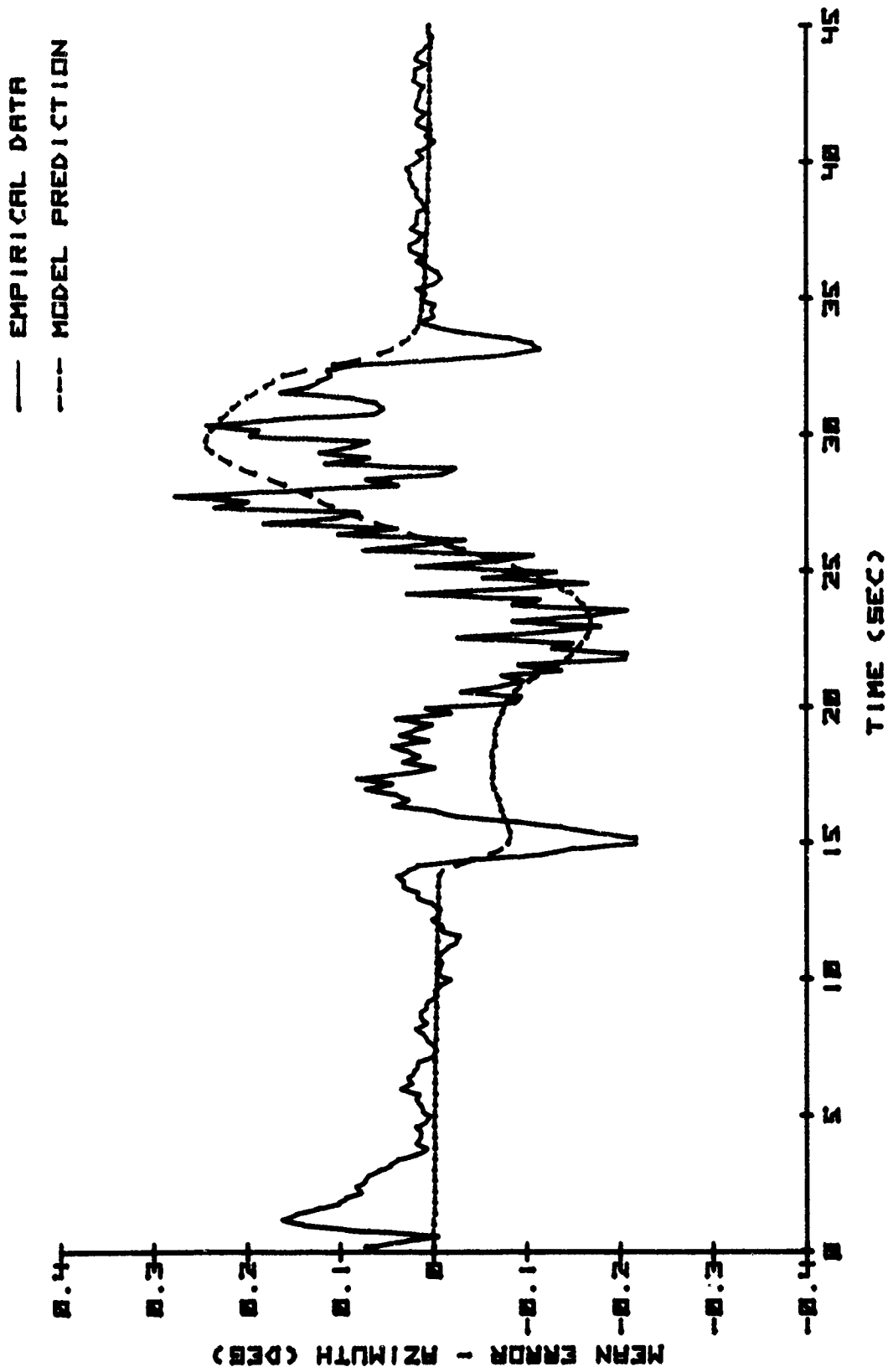


Figure 13. Mean Tracking Error Trajectory 3

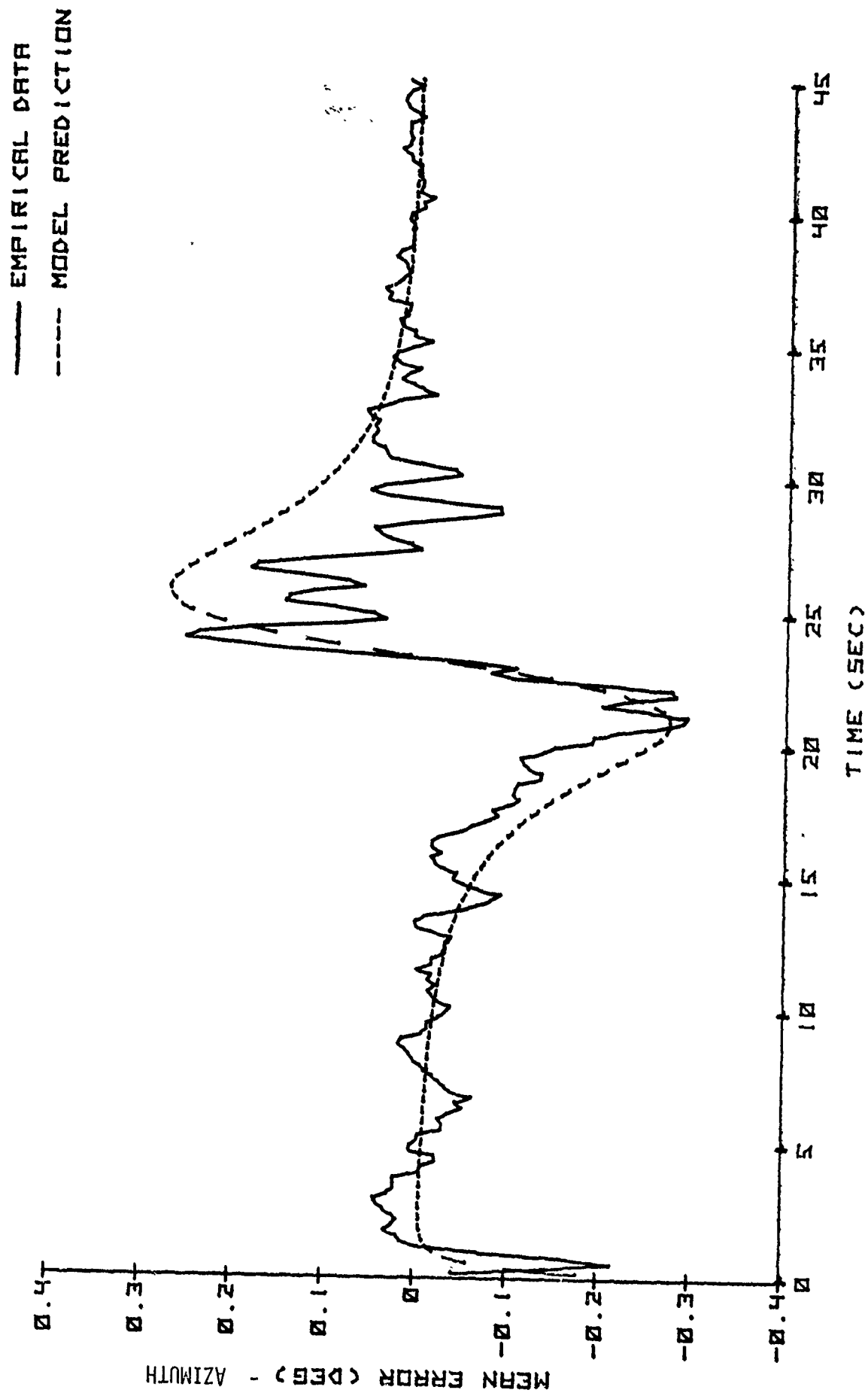


Figure 14. Mean Tracking Error Trajectory 4

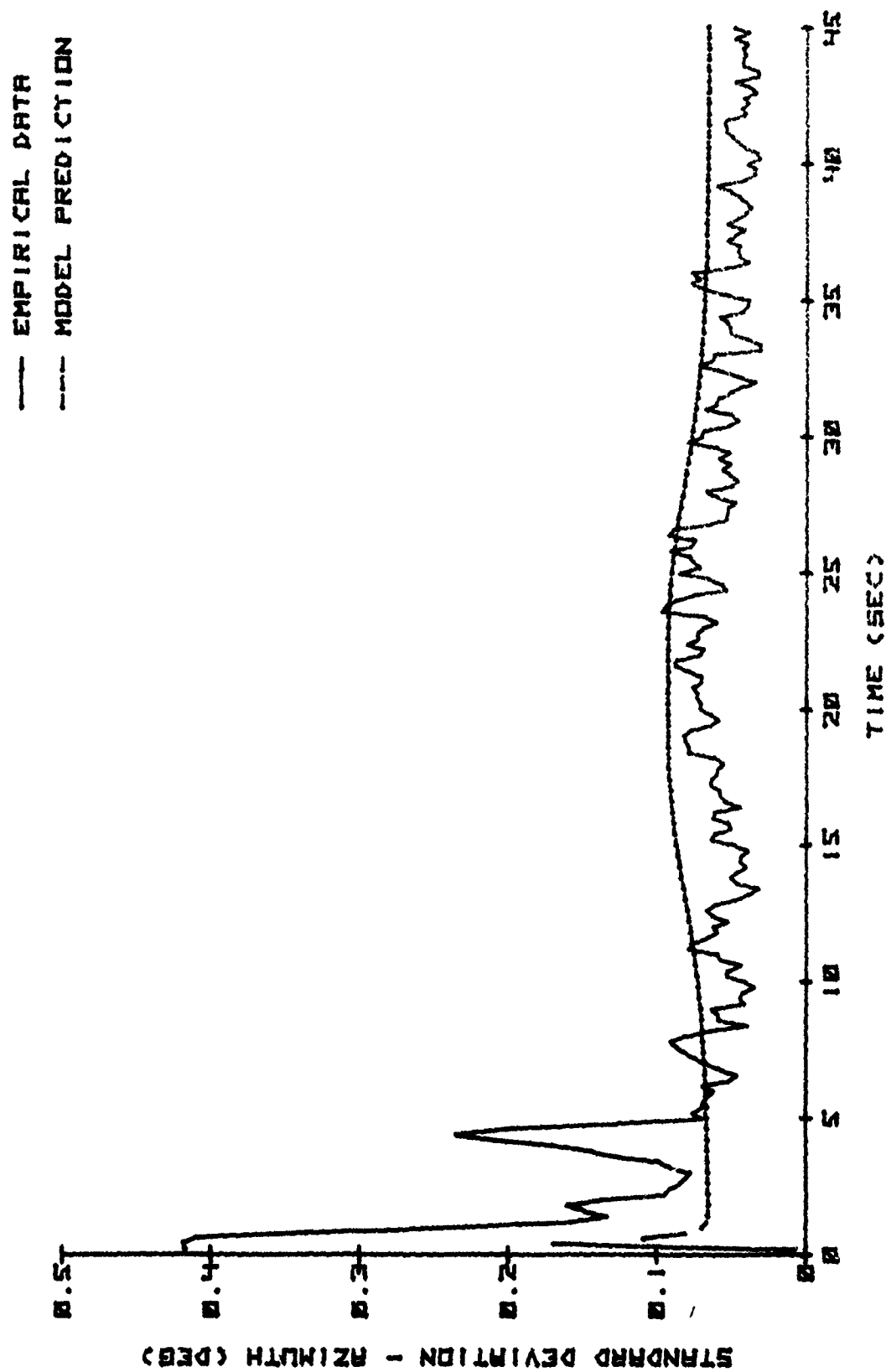


Figure 15. Standard Deviation of Tracking Error Trajectory 1

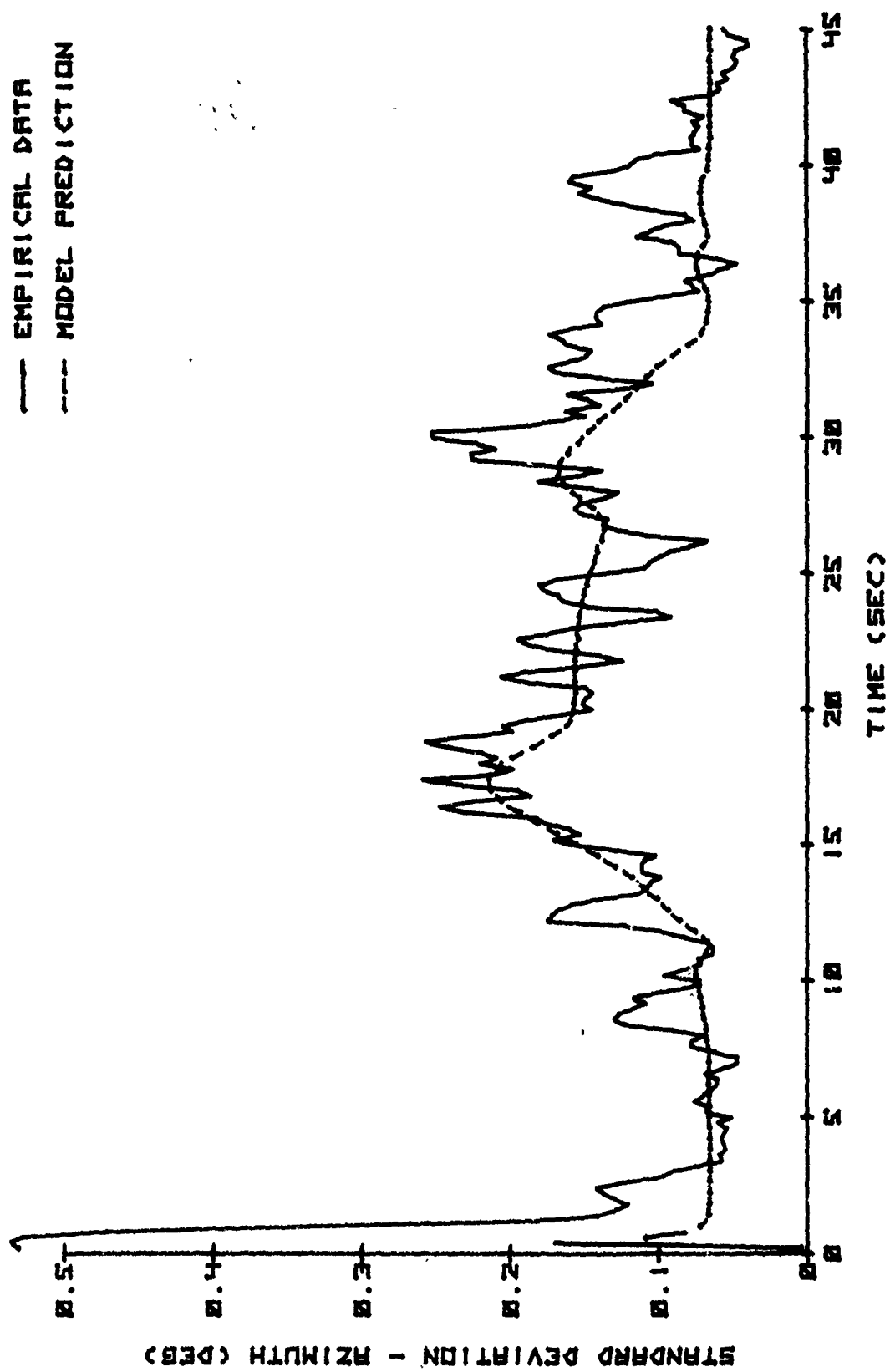


Figure 16. Standard Deviation of Tracking Error Trajectory 2

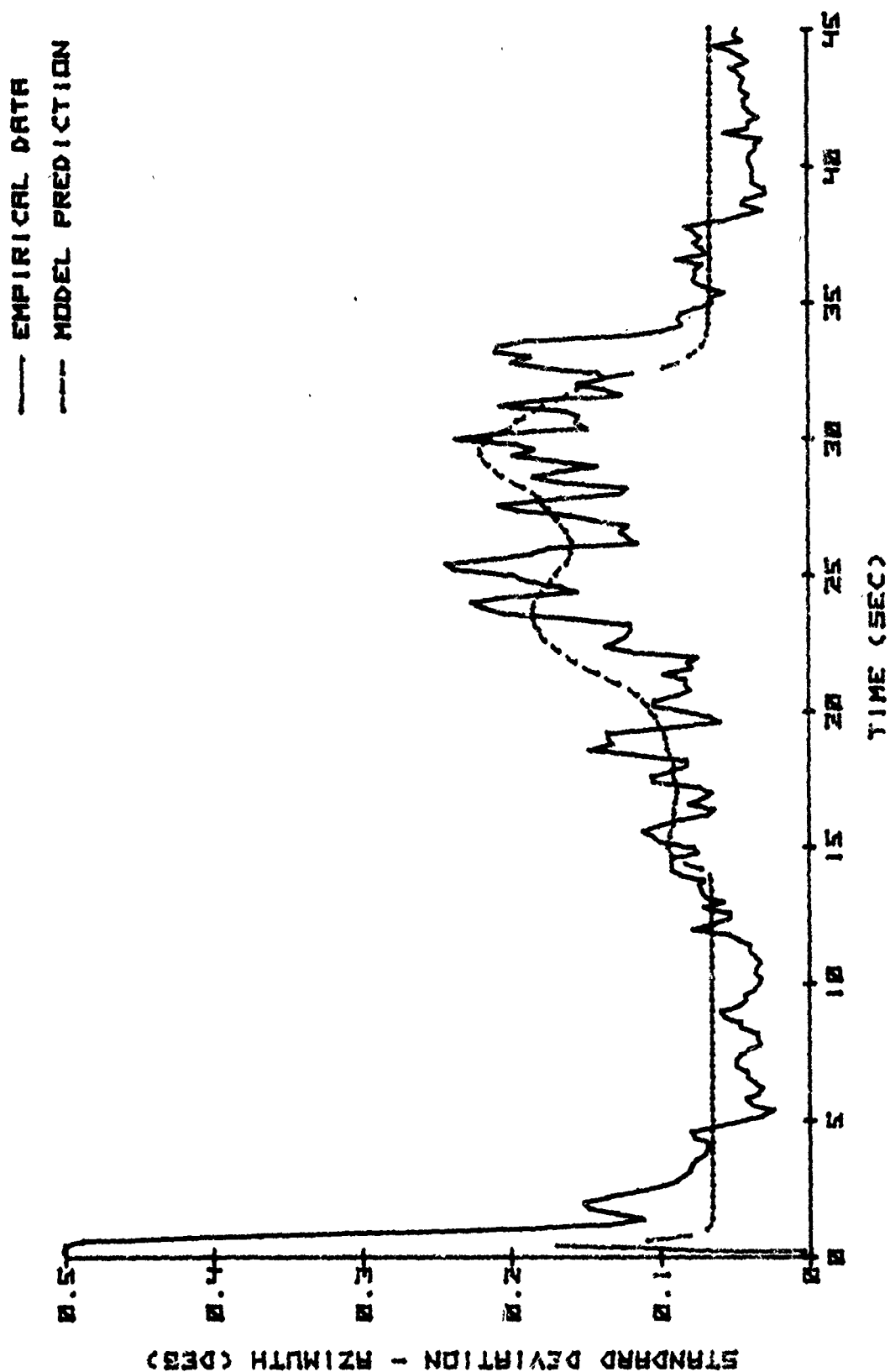


Figure 17. Standard Deviation of Tracking Error Trajectory 3

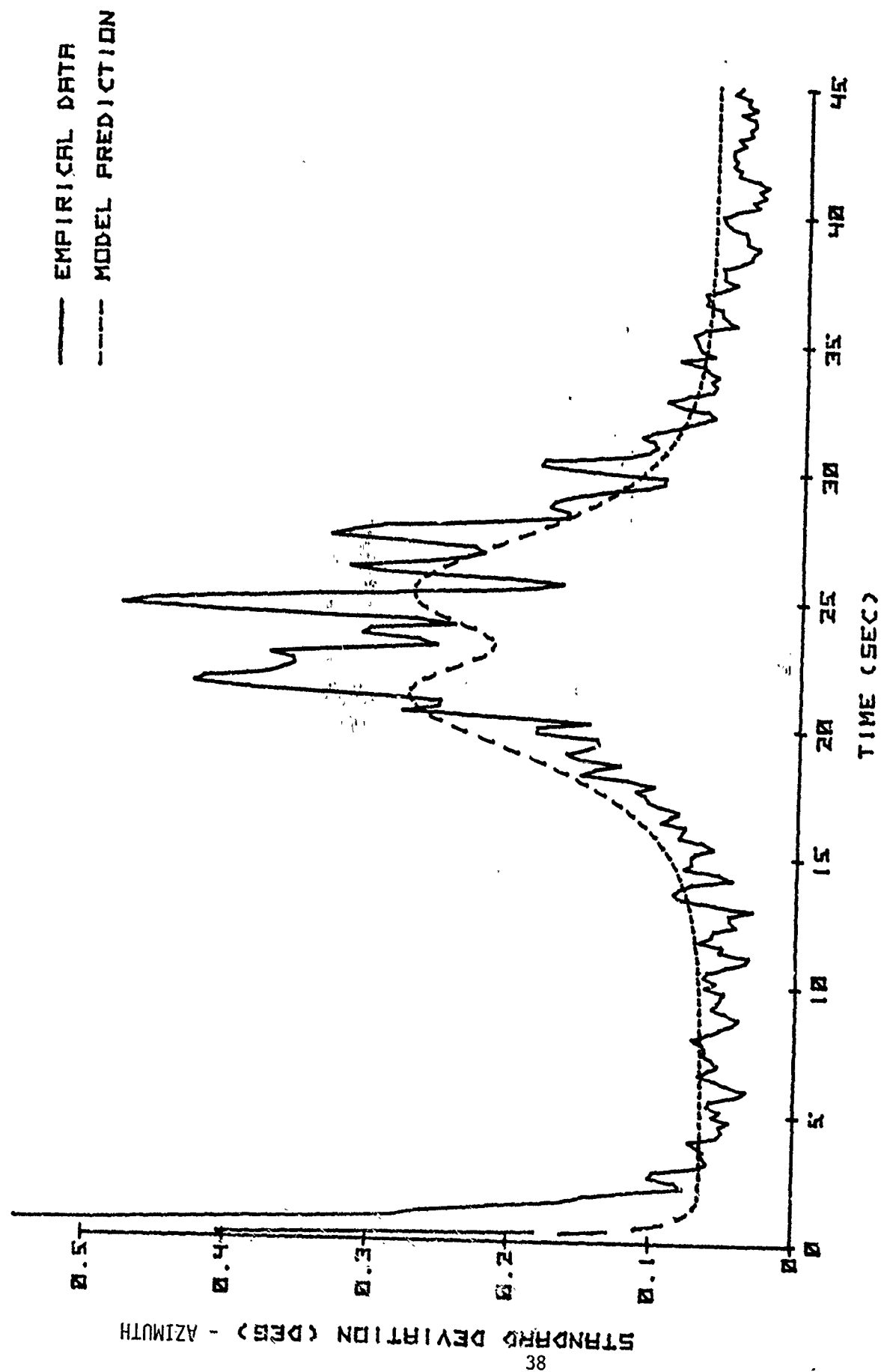


Figure 18. Standard Deviation of Tracking Error Trajectory 4

— EMPERICAL DATA  
 --- MODEL PREDICTION

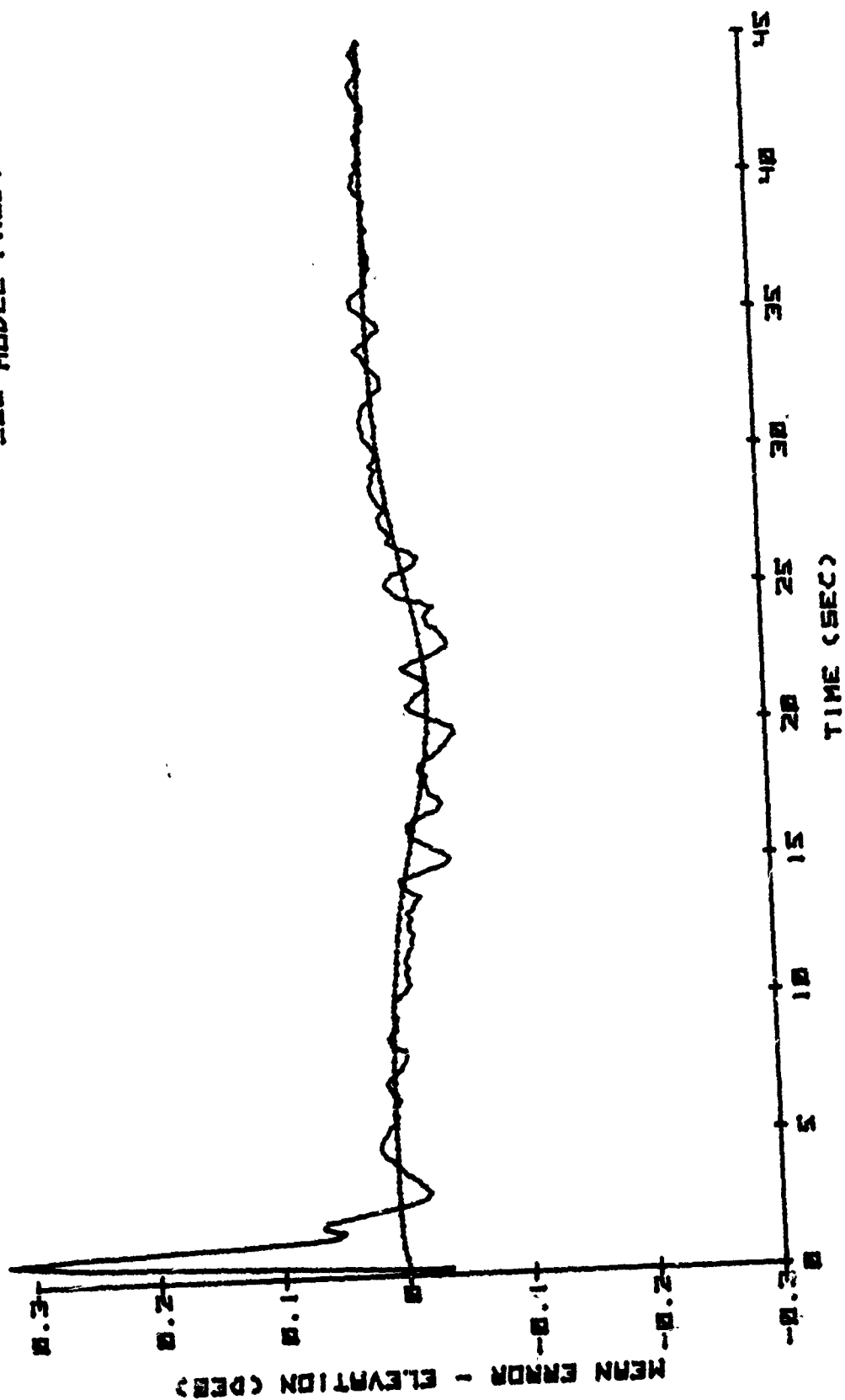


Figure 19. Mean Tracking Error Trajectory 1

— EMPNICAL DATA  
 --- MODEL PREDICTION

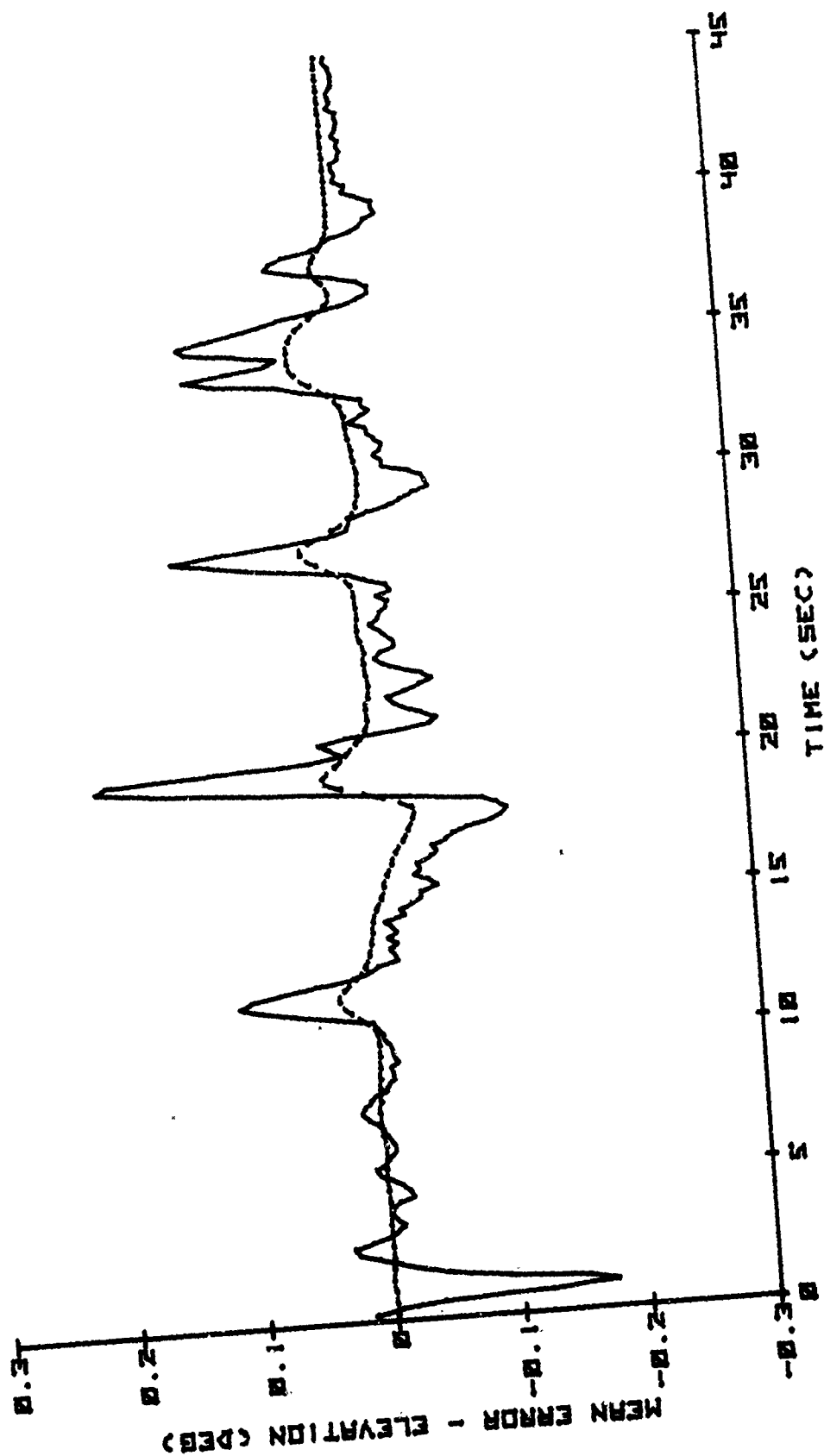


Figure 20. Mean Tracking Error Trajectory 2

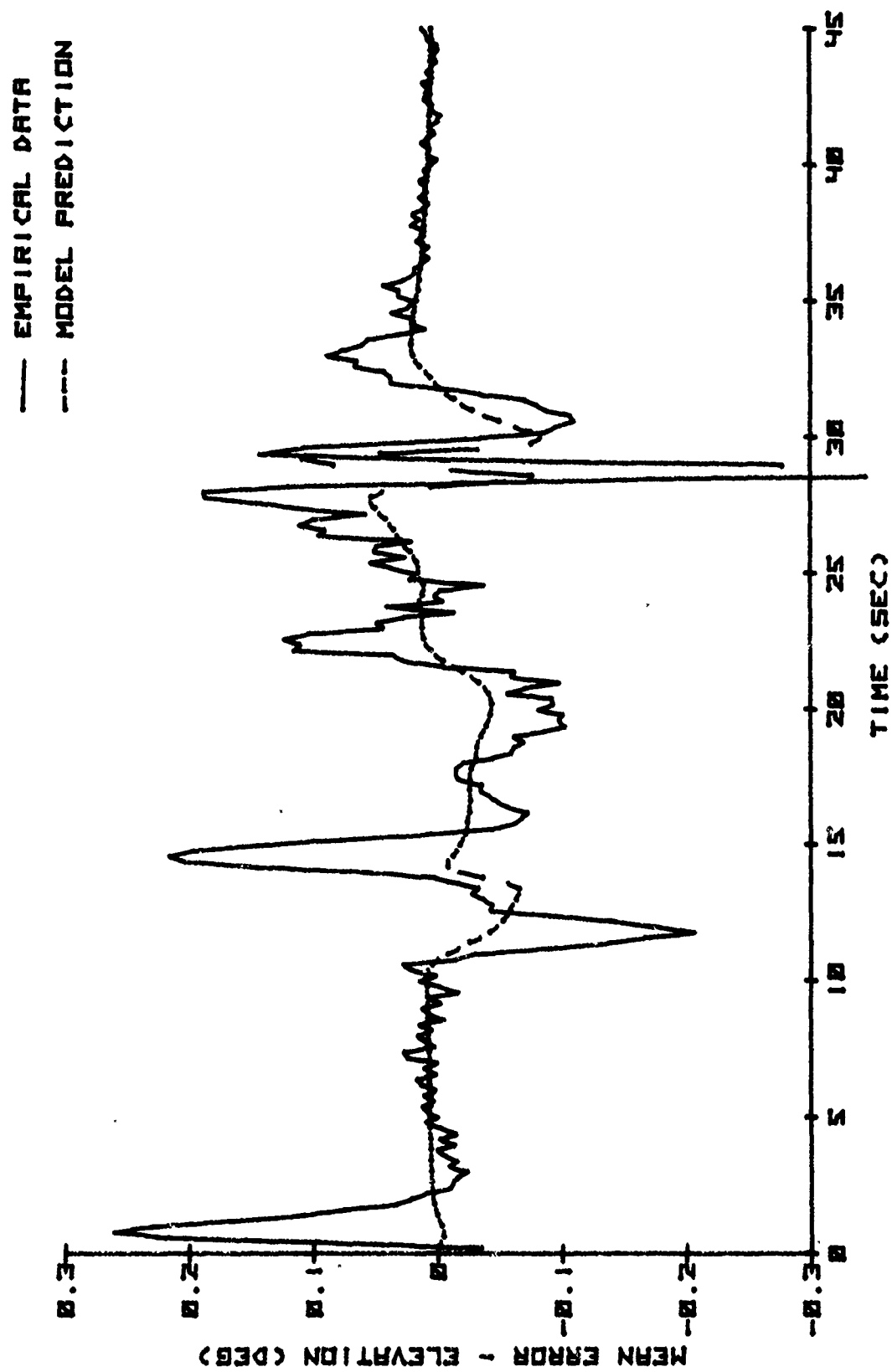


Figure 21. Mean Tracking Error Trajectory 3

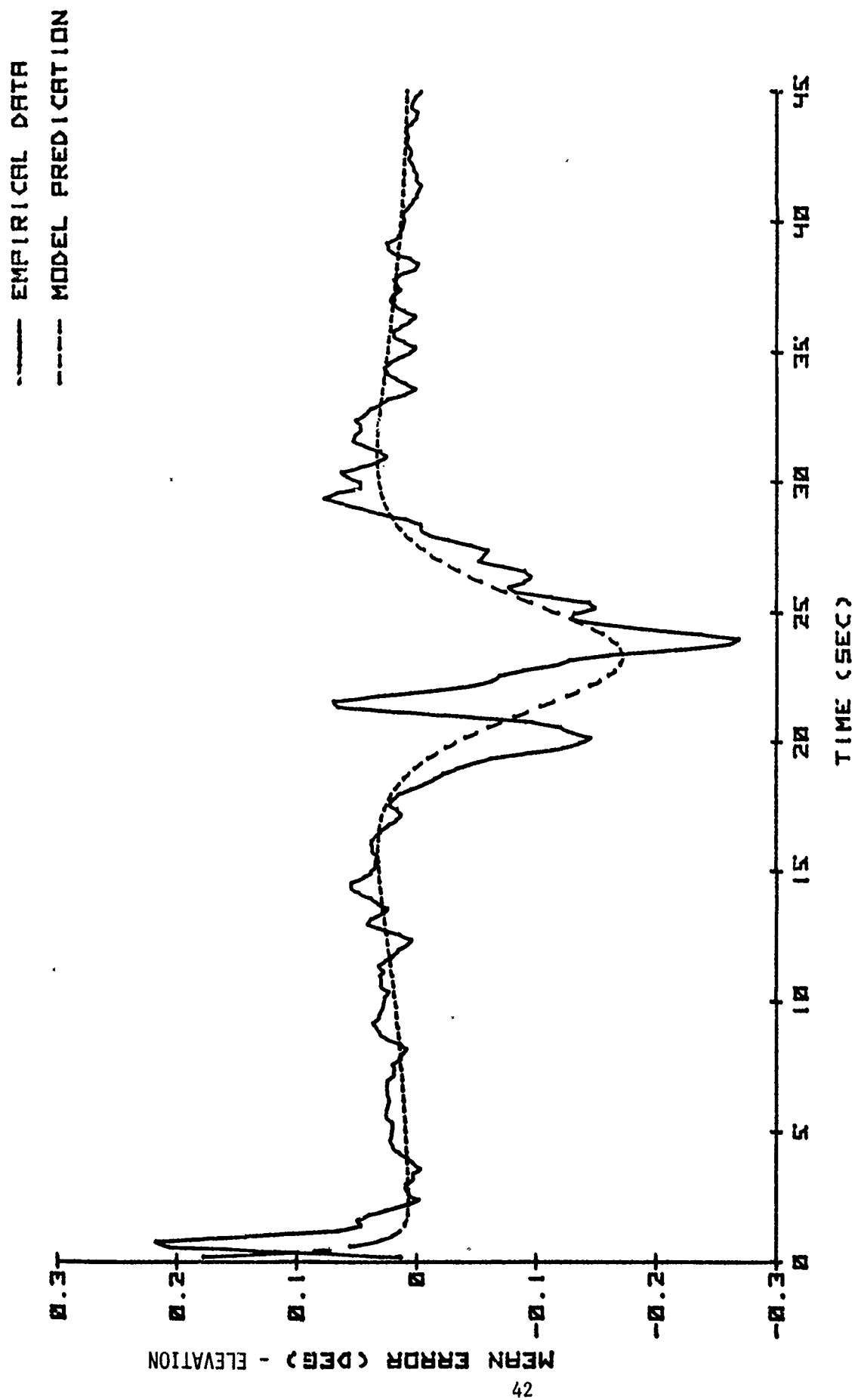


Figure 22. Mean Tracking Error Trajectory 4

— EMPIRICAL DATA  
--- MODEL PREDICTION

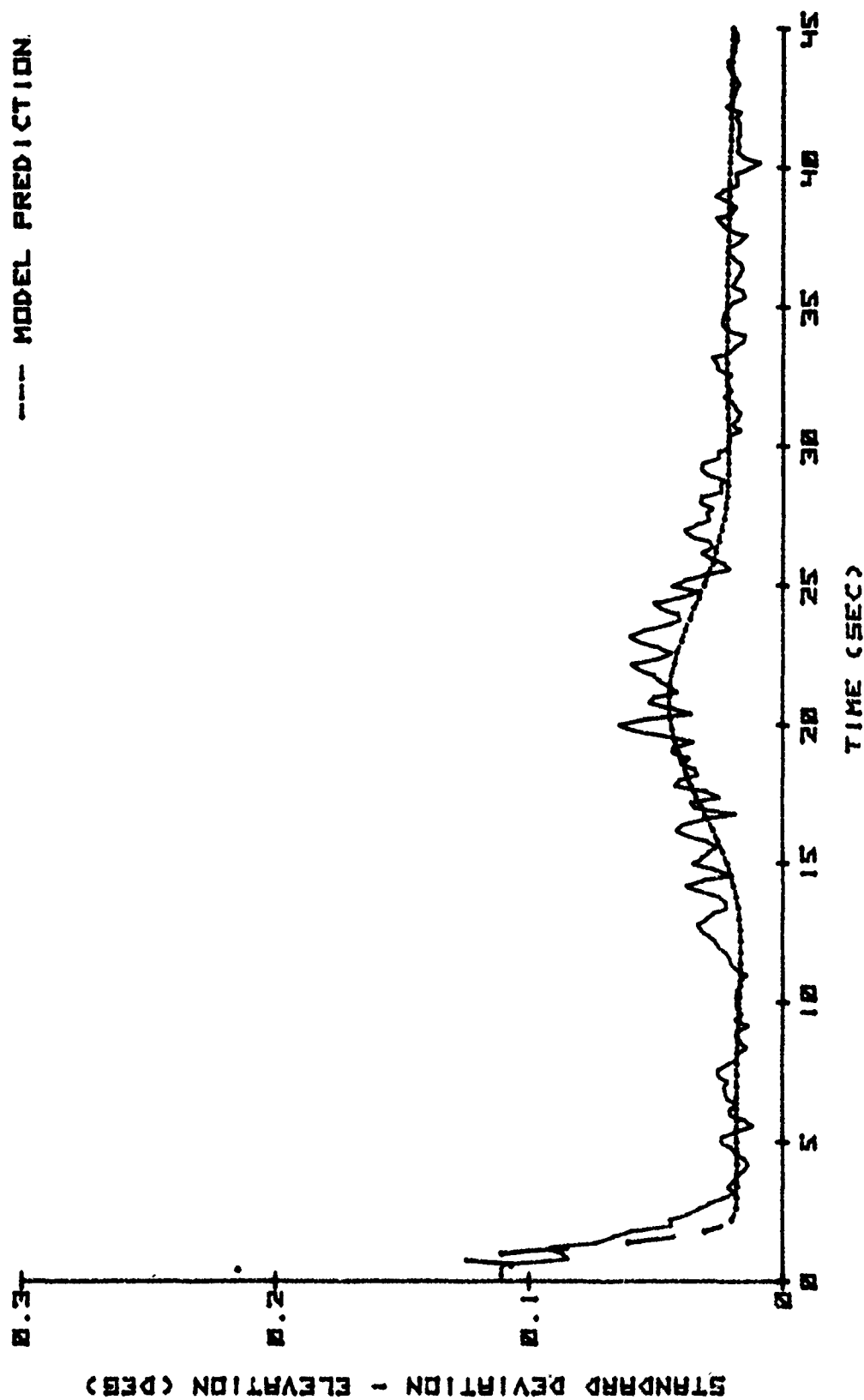


Figure 23. Standard Deviation of Tracking Error Trajectory 1

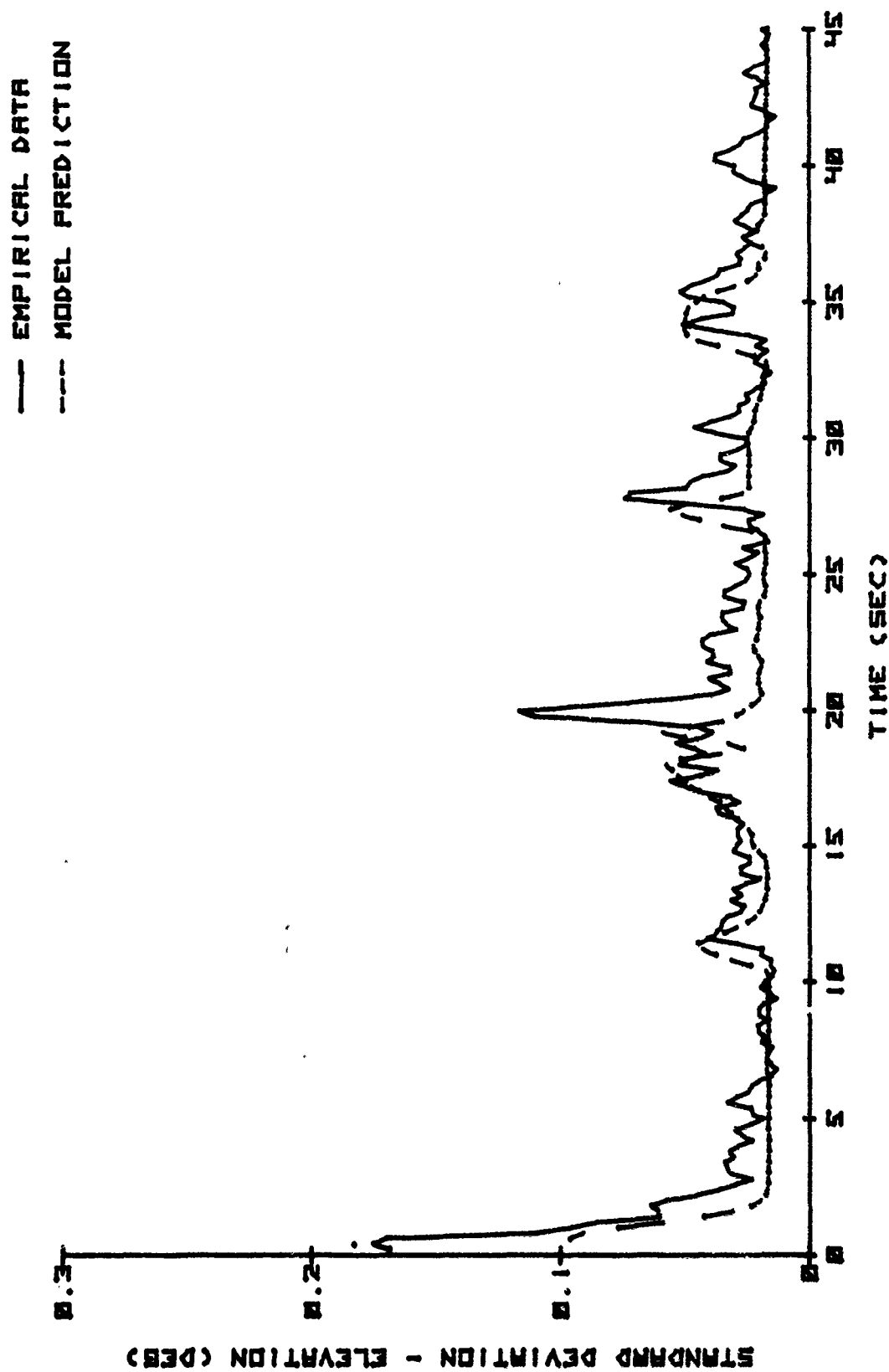


Figure 24. Standard Deviation of Tracking Error Trajectory 2

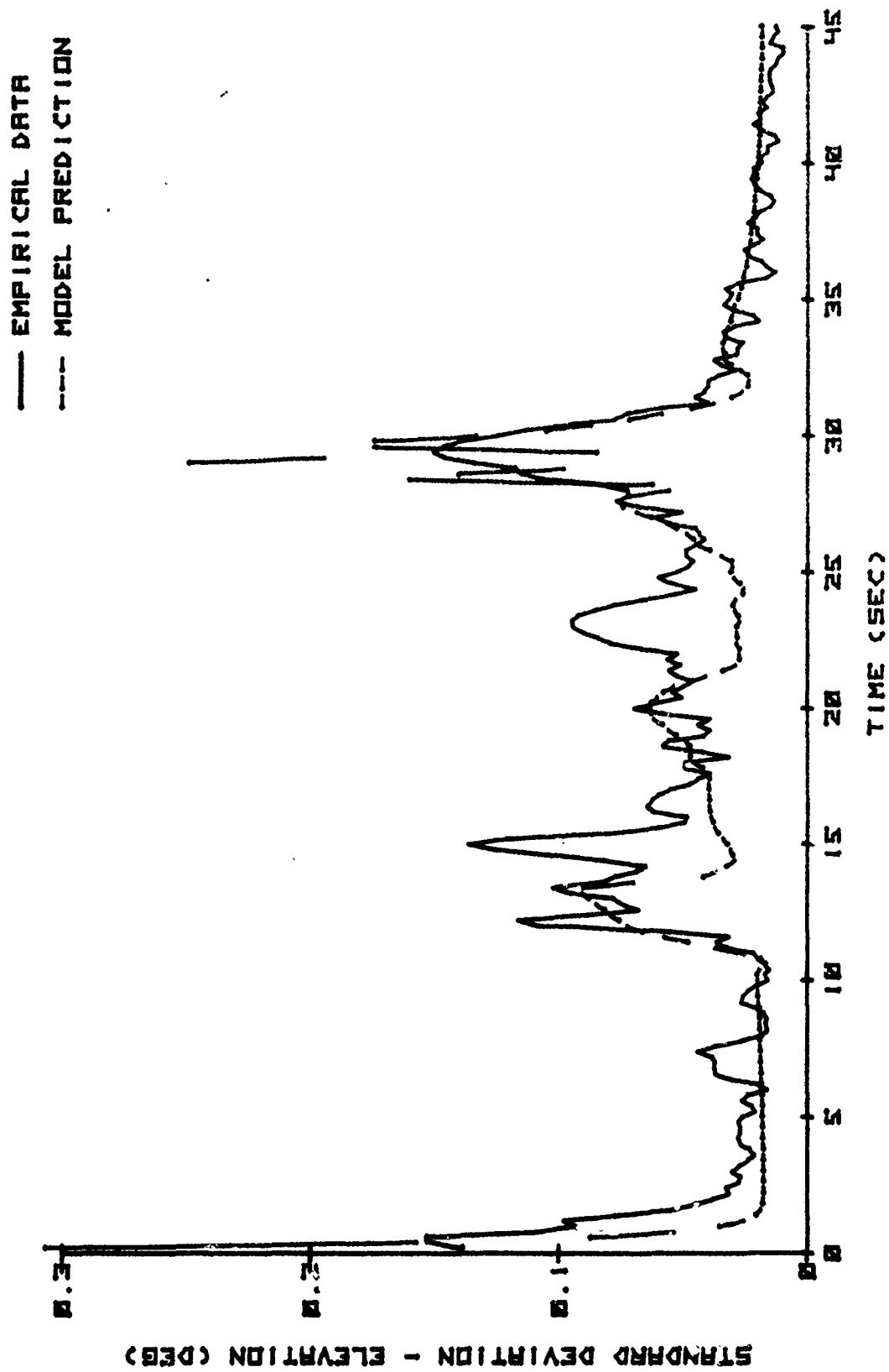


Figure 25. Standard Deviation of Tracking Error Trajectory 3

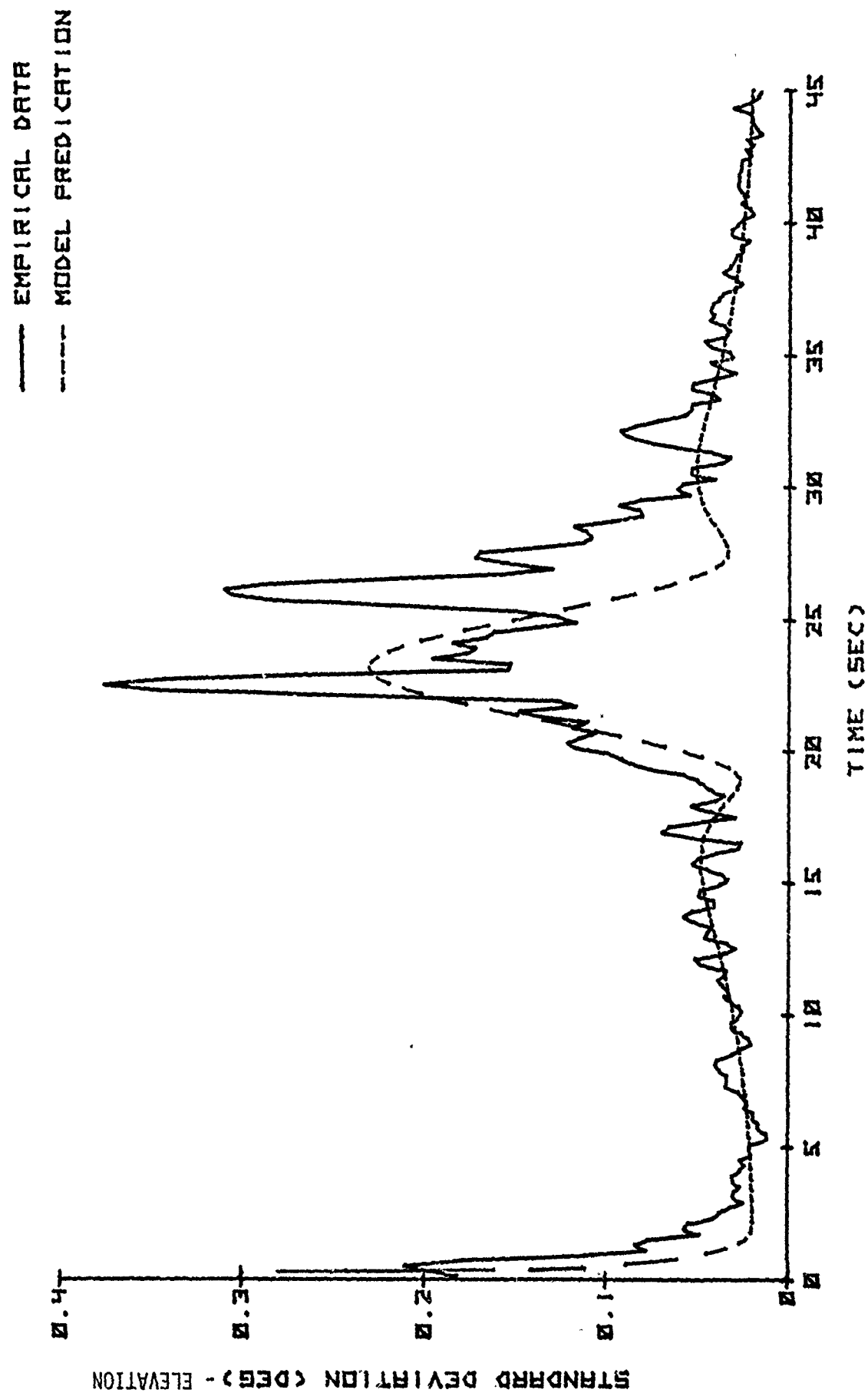


Figure 26. Standard Deviation of Tracking Error Trajectory 4

the ensemble standard deviation of azimuth tracking errors for four trajectories. Again, excellent matching between the two curves are shown in each of these four figures. Similar results of the elevation case are shown in Figures 19 through 26 for the four trajectories. All these results indicate that the gunner model is able to represent the characteristics of the gunner response in the AAA compensatory tracking task.

### C. Comparison

A comparison of the model prediction accuracy between this gunner model and the optimal control model has been done for several target trajectories. All the results show that both models give accurate predictions of tracking errors. Some typical results are shown in Figures 27 through 30. It is obvious that the gunner model developed by the authors can predict the tracking errors as accurately as those obtained by the optimal control model. However, the computer execution time for simulating the AAA gun system using the gunner model is less than 15% of that used by the optimal control model. It is a primary advantage of a model with simple structure. The following table shows some highlights of the gunner model over the optimal control model.

	Gunner Model (Observer Theory)	Optimal Control Model
Model Structure Complexity	Low	High
Computer Simulation Time (Seconds)	5.48	37.02
Model Predictions of Tracking Errors	Good	Good
Model Validation	Parameter Identification Program	Trial-and-Error Tuning

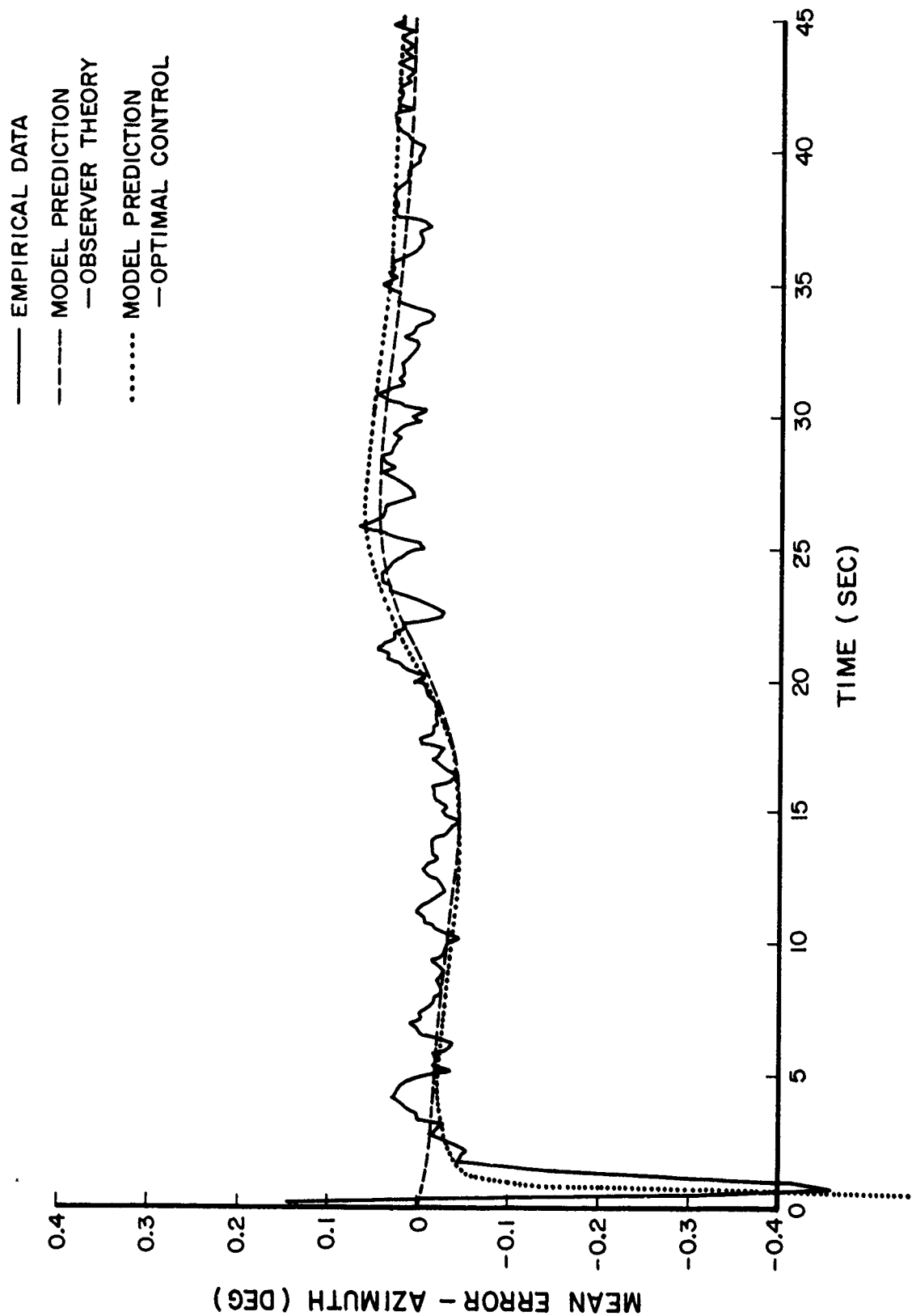


Figure 27. Comparison of Model Mean Predictions Trajectory 1

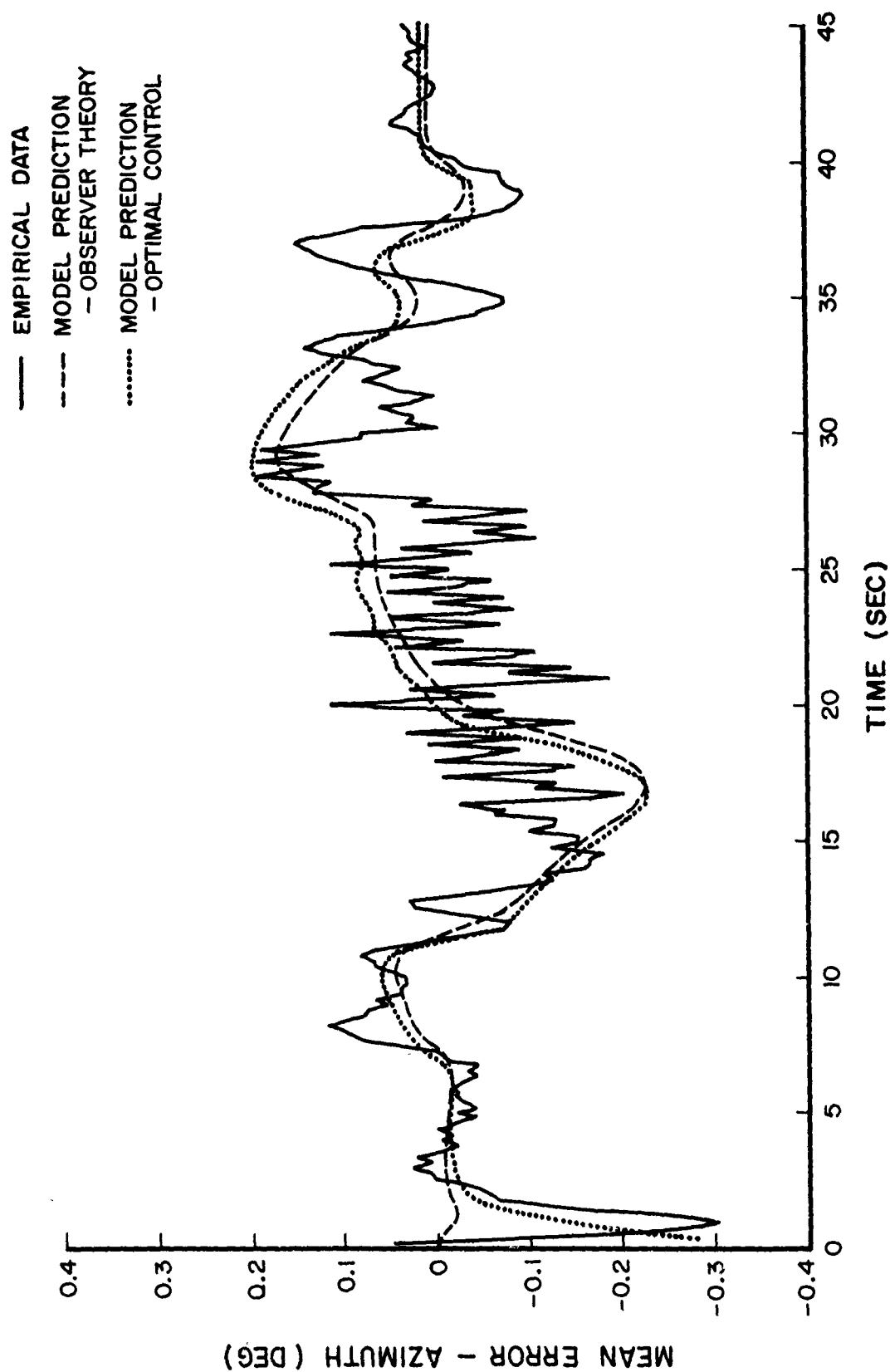


Figure 28. Comparison of Model Mean Predictions Trajectory 2

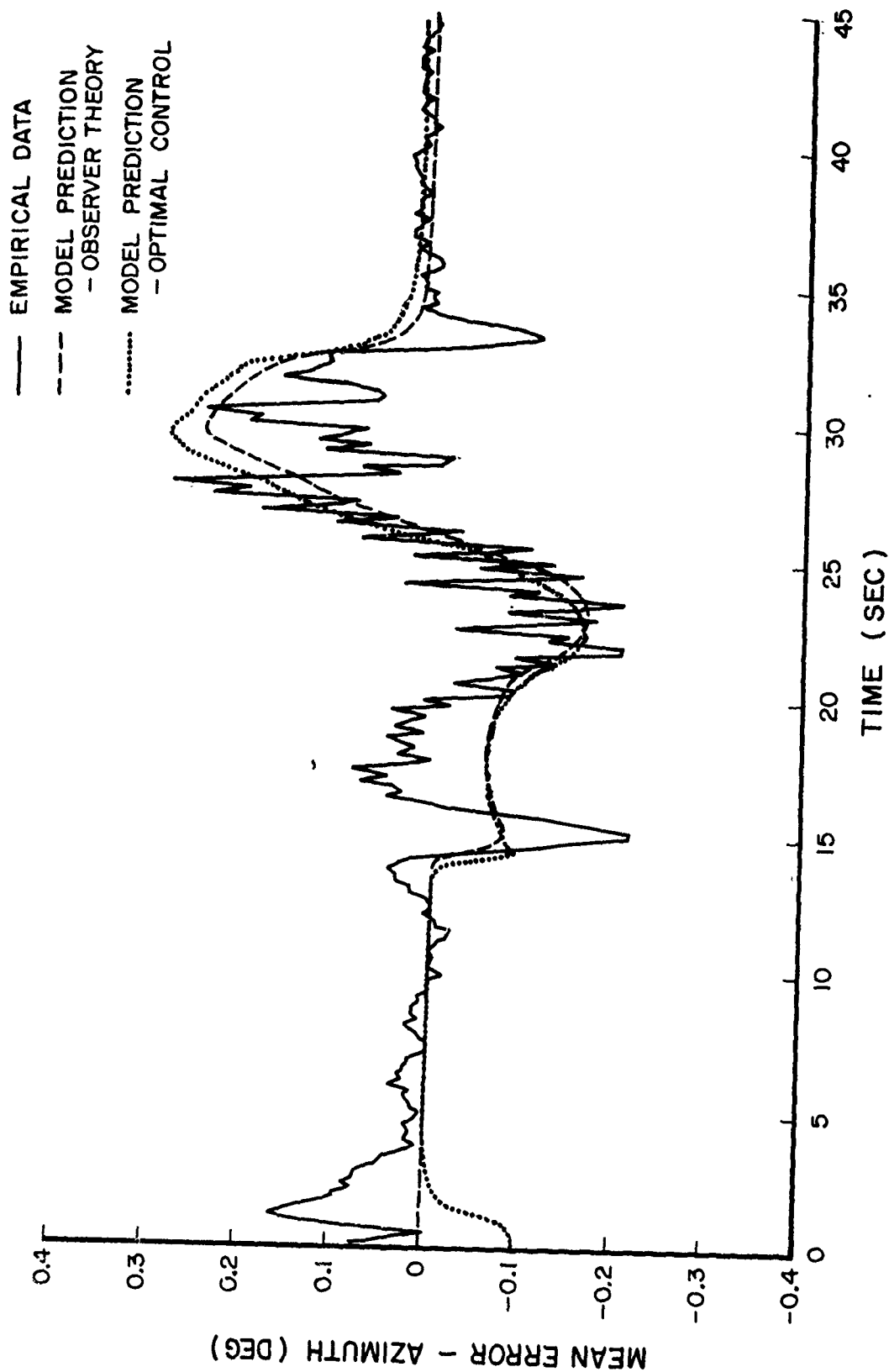


Figure 29. Comparison of Model Mean Predictions Trajectory 3

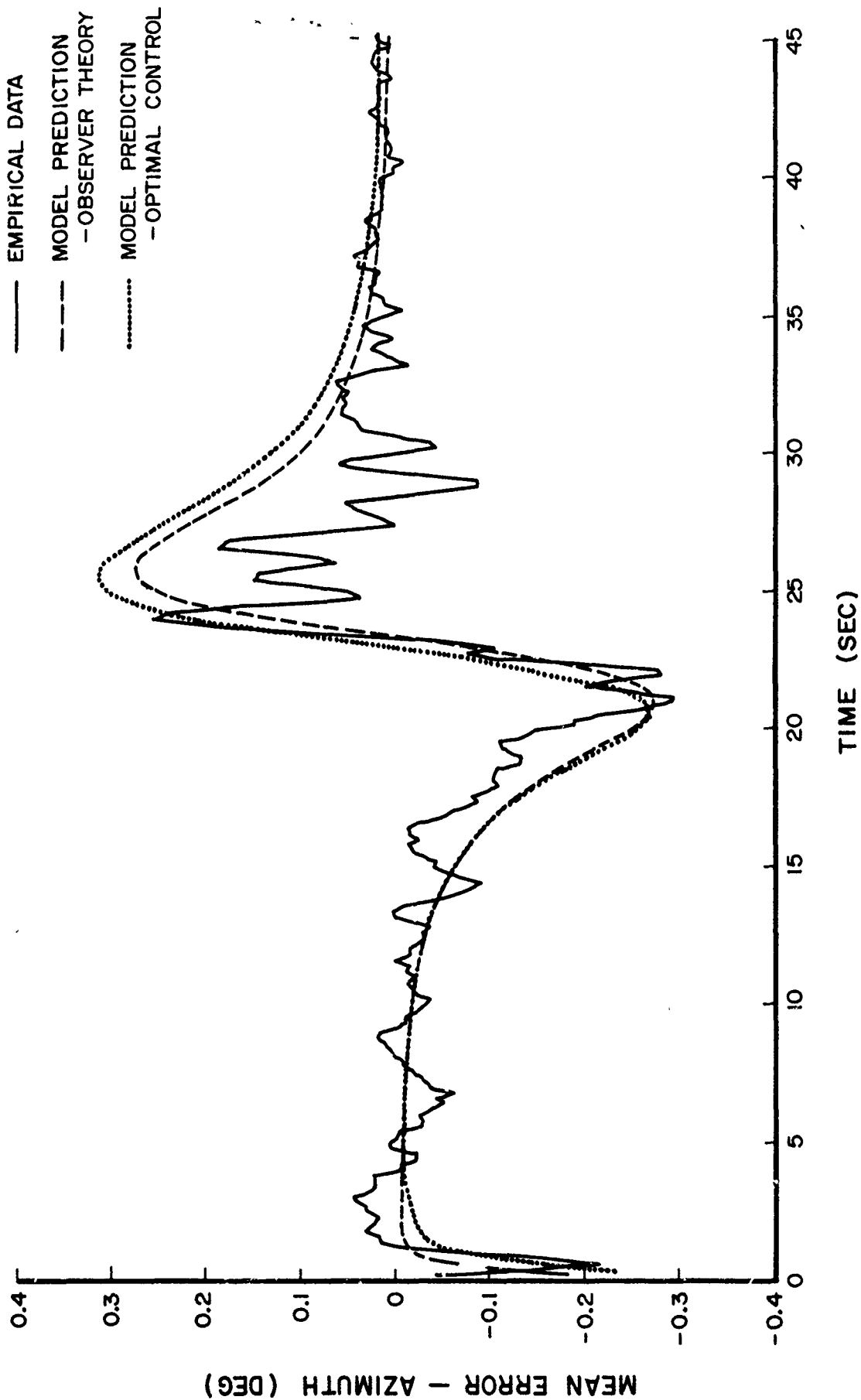


Figure 30. Comparison of Model Mean Predictions Trajectory 4

## Section IV

### CONCLUSION

The Luenberger reduced-order observer theory has been applied to design an antiaircraft gunner model which is composed of a reduced-order observer, a state variable feedback controller and a remnant element. The highlights of this model are simple structure and accurate model predictions. The key design requirement is to make the model structure simple so that it will shorten computer simulation time. It has also been shown in Figures 11 through 30 that this model can predict the tracking errors accurately. In addition, parameter identification program based on the least squares curve-fitting method and the Gauss Newton algorithm has been developed. It provides a systematic procedure to determine the numerical values of the model parameters. This gunner model has been used to study the AAA effectiveness of several foreign air defense weapon systems at the Aerospace Medical Research Laboratory, Wright-Patterson AFB. All the results show that it is an accurate and efficient antiaircraft gunner model. Therefore, this model can be used to study AAA effectiveness and aircraft survivability.

## APPENDIX A

## APPENDIX A

### Derivation of Model Prediction of Mean Tracking Error

The model prediction  $\bar{e}_T'$  of the ensemble mean of the tracking error as an explicit function of time  $t$  and parameters  $\gamma_1$ ,  $\gamma_2$ , and  $k$  is derived in this Appendix. First rewrite Eq. (16)

$$\dot{\bar{X}} = A_1 \bar{X} + F_1 \ddot{\theta}_T$$

It can be shown that the solution  $\bar{X}$  of Eq. (16) is given by

$$\bar{X}(t) = \int_{-\infty}^t \phi(t - \tau) F_1 \ddot{\theta}_T(\tau) d\tau$$

where  $\phi$  is a  $3 \times 3$  matrix and equals:

$$\phi(t) = e^{A_1 t} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix}$$

The first component  $\bar{x}_1(t)$  represents the ensemble mean of the tracking error. From Eqs. (1) and (15),

$$F_1 = \begin{bmatrix} f_1 \\ f_2 - kf_1 \\ f_2 - kf_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\bar{e}'_T(t)$  can be expressed by

$$\bar{e}'_T = \bar{x}_1(t) = \int_{-\infty}^t \left[ \phi_{12}(t - \tau) + \phi_{13}(t - \tau) \right] \ddot{\theta}_T(\tau) d\tau$$

Let  $G(s)$  be a  $3 \times 3$  matrix which represents the Laplace transform of the transition matrix  $\phi(t)$ ; then it can be shown that

$$G(s) = (sI - A_1)^{-1} = \frac{\text{Adj}(sI - A_1)}{\det(sI - A_1)}$$

where  $s$  is the variable of the complex plane and  $I$  is a  $3 \times 3$  identity matrix.  $\det$  and  $\text{Adj}$  denote the determinant and adjoint of a matrix respectively. Let  $G_{ij}(s)$  be the  $ij$ th element of the matrix  $G(s)$ ; then

$$G_{12}(s) = \frac{1 + \gamma_2}{s^2 - \gamma_1 s}$$

and

$$G_{13}(s) = \frac{-\gamma_2}{s^2 + (k - \gamma_1)s - k\gamma_1}$$

For simplicity, select  $\gamma_2 = -1$ , then  $G_{12}(s) = 0$  and

$$G_{13}(s) = \frac{1}{s^2 + (k - \gamma_1)s - k\gamma_1}$$

Taking the inverse Laplace transform of  $G_{13}(s)$ , we have

$$\phi_{13}(t) = \frac{e^{\gamma_1 t} - e^{-kt}}{\gamma_1 + k}$$

So

$$\bar{e}'_T(t) = \int_{-\infty}^t \frac{e^{\gamma_1(t-\tau)} - e^{-k(t-\tau)}}{\gamma_1 + k} \ddot{\theta}_T(\tau) d\tau$$

This is the explicit function of  $\bar{e}'_T$  in terms of the unknown parameters.

The derivatives of  $\bar{e}'_T(t)$  with respect to parameters  $k$  and  $\gamma_1$  are expressed in the following

$$\frac{\partial \bar{e}'_T(t)}{\partial k} = \int_{-\infty}^t \frac{(\gamma_1 + k)(t - \tau)e^{-k(t-\tau)} - (e^{\gamma_1(t-\tau)} - e^{-k(t-\tau)})}{(\gamma_1 + k)^2} \cdot \ddot{\theta}_T(\tau) d\tau$$

$$\frac{\partial \bar{e}'_T(t)}{\partial \gamma_1} = \int_{-\infty}^t \frac{(\gamma_1 + k)(t - \tau)e^{\gamma_1(t-\tau)} - (e^{\gamma_1(t-\tau)} - e^{-k(t-\tau)})}{(\gamma_1 + k)^2} \cdot \ddot{\theta}_T(\tau) d\tau$$

These quantities are used in the mean curve-fitting parameter identification program.

## APPENDIX B

### Derivation of Model Prediction of Standard Deviation of Tracking Error

The function  $S_2$  which is the square of the standard deviation of the tracking error is derived in terms of  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ . Equation (18) is rewritten here:

$$P(t) = \phi(t, t_0) P(t_0) \phi^T(t, t_0) + \int_{t_0}^t \phi(t, \tau) D_1 q(\tau) D_1^T \phi^T(t, \tau) d\tau$$

where

$$q(\tau) = \alpha_1 + \alpha_2 \hat{\theta}_T^2(\tau) + \alpha_3 \hat{\theta}_T^2(\tau),$$

$$D_1 = \begin{bmatrix} b_1 \\ b_2 - kb_1 \\ b_2 - kb_1 \end{bmatrix} = \begin{bmatrix} -1 \\ k \\ k \end{bmatrix}$$

and let

$$Q(\tau) = D_1 q(\tau) D_1^T = \begin{bmatrix} 1 & -k & -k \\ -k & k^2 & k^2 \\ -k & k^2 & k^2 \end{bmatrix} \cdot q(\tau)$$

Since the first element  $p_{11}$  in the diagonal of matrix  $P(t)$  is the square of the standard deviation of the tracking error and denoted by  $S_2$ , then,

$$S_2 = p_{11}(t) = \sum_{i=1}^3 \phi_{1i}^2(t, t_0) p_{ii}(t_0) + \int_{t_0}^t \sum_{j=1}^3 \sum_{k=1}^3$$

$$\phi_{1k}(t-\tau) q_{kj}(\tau) \phi_{1j}(t-\tau) d\tau$$

where  $q_{kj}$  denotes the  $kj$ th element of the matrix  $Q$ . As shown in Appendix A, the parameter  $\gamma_2$  is selected to equal -1. Therefore  $\phi_{12}(t) = 0$ . Then

$$S_2(t) = \phi_{11}^2(t, t_0) p_{11}(t_0) + \phi_{13}^2(t, t_0) p_{33}(t_0) + \int_{t_0}^t [\phi_{11}^2(t-\tau) + \phi_{13}^2(t-\tau) k^2 - 2 \phi_{11}(t-\tau) \phi_{13}(t-\tau) k] q(\tau) d\tau$$

where  $\phi_{13}(t)$  has been computed in Appendix A and  $\phi_{11}(t)$  is obtained in the following. From Appendix A, it can be shown that

$$G_{11}(s) = \frac{1}{s - \gamma_1}$$

So we have

$$\phi_{11}(t) = e^{\gamma_1 t}$$

Then

$$S_2(t) = e^{2\gamma_1(t-t_0)} p_{11}(t_0) + \left( \frac{e^{\gamma_1(t-t_0)} - e^{-k(t-t_0)}}{\gamma_1 + k} \right)^2 p_{33}(t_0) +$$

$$\int_{t_0}^t \left[ e^{\gamma_1(t-\tau)} - k \frac{e^{\gamma_1(t-\tau)} - e^{-k(t-\tau)}}{\gamma_1 + k} \right]^2$$

$$\cdot \left( \alpha_1 + \alpha_2 \hat{\theta}_T^2(\tau) + \alpha_3 \hat{\theta}_T^2(\tau) \right) d\tau$$

This is the explicit function of  $S_2$  in terms of  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ .

Let

$$f_1(t) = e^{2\gamma_1(t-t_0)} p_{11}(t_0) + \left( \frac{e^{\gamma_1(t-t_0)} - e^{-k(t-t_0)}}{\gamma_1 + k} \right)^2 p_{33}(t_0)$$

and

$$f_2(t) = \left( e^{\gamma_1 t} - k \frac{e^{\gamma_1 t} - e^{-kt}}{\gamma_1 + k} \right)^2$$

then

$$S_2(t) = f_1(t) + \int_{t_0}^t f_2(t-\tau) \left( \alpha_1 + \alpha_2 \hat{\theta}_T^2(\tau) + \alpha_3 \hat{\theta}_T^2(\tau) \right) d\tau$$

Now the derivations of  $S_2(t)$  with respect to  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are

$$\frac{\partial S_2(t)}{\partial \alpha_1} = \int_{t_0}^t f_2(t-\tau) d\tau$$

$$\frac{\partial S_2(t)}{\partial \alpha_2} = \int_{t_0}^t f_2(t-\tau) \hat{\theta}_T^2(\tau) d\tau$$

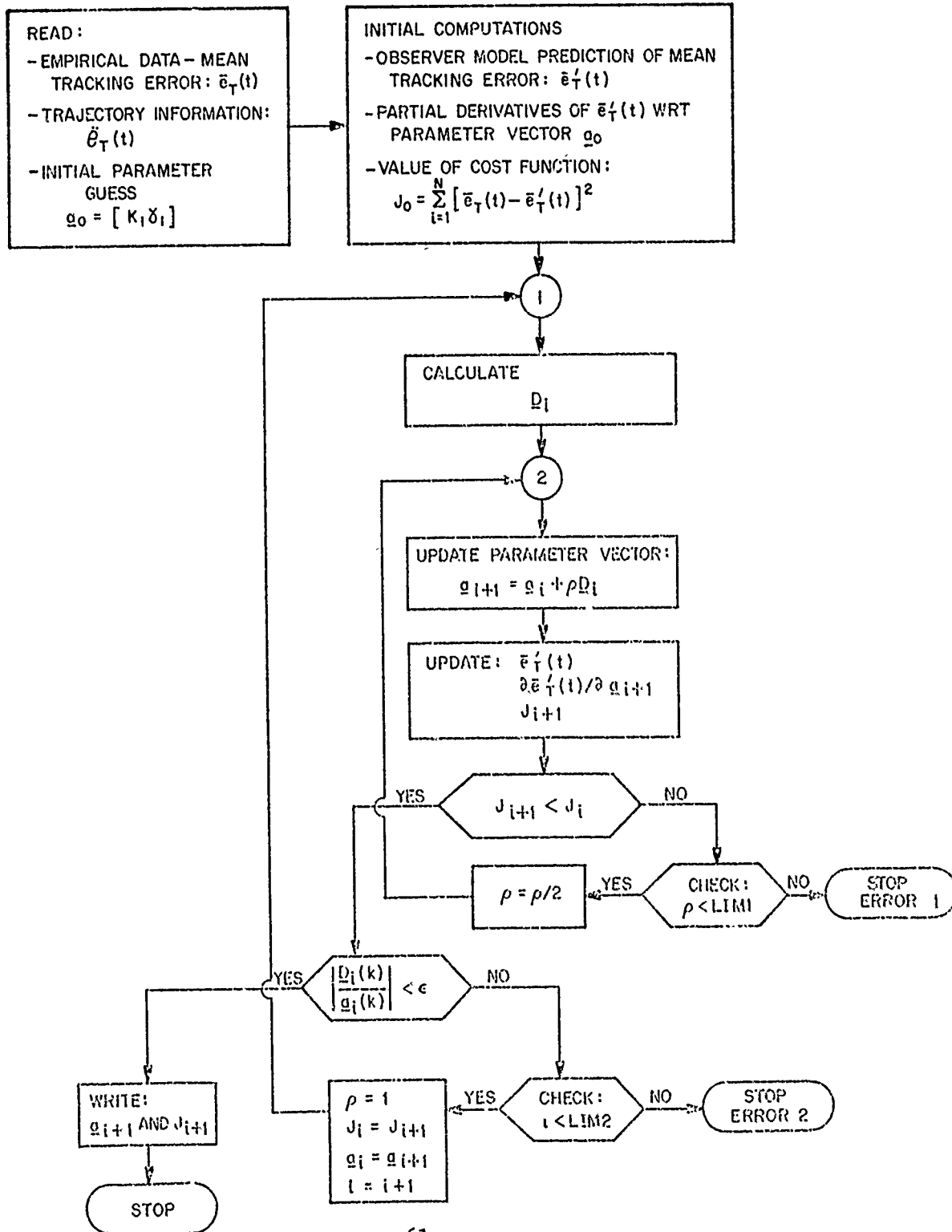
$$\frac{\partial S_2(t)}{\partial \alpha_3} = \int_{t_0}^t f_2(t-\tau) \hat{\theta}_T^2(\tau) d\tau$$

These quantities are used in the standard deviation curve-fitting parameter identification program.

# APPENDIX C

## Flow Chart and Program Listing of Time Domain Curve-Fitting Program

### A. Flow Chart of Parameter Identification for Parameters Associated with Mean Tracking Error Equation.



Comments: Execution of Program Fit.

1. Compile Program
2. Input: Tape 1 contains
  - a. Azimuth and elevation trajectory information
  - b. Empirical data - mean tracking error
3. Load Library Routines:
  - a. VCONVO - convolution integral computation from International Mathematical & Statistical Library (IMSL).
  - b. GMINV - matrix inverse routine from D. L. Kleinman Library [12].
4. Execute
5. Output: Parameter values  $[k, \gamma_1]$ .

```

PROGRAM FIT(INPUT,OUTPUT,TAPE1)
COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL,N1,N2
COMMON/MAT/F2(1024),A(2),AA(2),B(1024,2),D(2),F3(4096)
DATA N,H,IP,N1,12/1024,.J4,2,100,1024/
C CURVE FIT ROUTINE TO DETERMINE IF=2 PARAMETERS KGAIN & GAMMA1
DO 10 I=1,N
10 READ(1,3)F3(I+N),Z,Z1,Z2
DO 20 I=1,N
20 READ(1,4)Z,Z1,F2(I),Z2,Z3,Z4
3 FORMAT(4G12.5)
4 FORMAT(5G12.4)
1 PRINT*,50H TYPE 1 TO GO OR TYPE 0 TO STOP
READ*,IFG
IF(IFG.EQ.J)STOP
CALL INIT
GO TO 1
END

SUBROUTINE INIT
C INITIALIZATION ROUTINE
C READ IN DATA TO BE FITTED
C MAKE INITIAL GUESS
COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL,N1,N2
COMMON/MAT/F2(1024),A(2),AA(2),B(1024,2),D(2),F3(4096)
PRINT*,30H TYPE IN LIM1,LIM2,EE
READ*,LIM1,LIM2,EE
PRINT*,3H N=,N,2HH=,H,5HLIM1=,LIM1,5HLIM2=,LIM2,3HEE=,EE,3HIP=,IP
PRINT*,50H TYPE IN INITIAL GUESS - KGAIN, GAMMA1
READ*,AA
PRINT*,10H 1ST GUESS,AA
DEL=1
CALL COEF(AA,RJ2)
PRINT*,7H JMIN1=,RJ2
CALL LOOP(RJ2)
END

SUBROUTINE DO
C CALCULATE J MATRIX FROM Q AND R
DIMENSION R(2),Q(2,2),W1(2,2)
COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL,N1,N2
COMMON/MAT/F2(1024),A(2),AA(2),B(1024,2),D(2),F3(4096)
COMMON/MAIN/NDIM,NDIM1
NCIM=IP;NDIM1=IP+1
DO 10 I=1,IP
R(I)=0.
DO 10 J=1,IP
10 Q(I,J)=0.
DO 35 K=N1,N2
SN=F3(K)-F3(K+N)
DO 30 I=1,IP
DO 25 J=1,IP
25 Q(I,J)=Q(I,J)+B(K,I)*B(K,J)
30 R(I)=R(I)+B(K,I)*SN
35 CONTINUE
CALL GMINV(IP,IP,Q,W1,MR,D)
IP1=IP*IP

```

```

DO 50 I=1,IP
  II=1
  D(I)=0.
DO 45 J=1,IP1,IP
  D(I)=D(I)+W1(J)*R(II)
45  II=II+1
50  D(I)=-D(I)
END
SUBROUTINE LOOP(RJ2)
C  ITERATION PROCESS
C  COMPUTE A(I+1)=A(I)+D(I)
C  DETERMINE WHEN A(I+1) IS ACCEPTABLE
  DIMENSION DDD(2)
  COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL,N1,N2
  COMMON/MAT/F2(1;24),A(2),AA(2),B(1024,2),D(2)
  NCT=MCT=0.
1  CALL DD
2  DO 100 I=1,IP
  D(I)=D(I)*DEL
100  A(I)=AA(I)+D(I)
  IF(A(1).LT.0..OR.A(2).GT.0.)GO TO 15
  CALL COEF(A,RJ1)
6  IF(RJ1.LT.RJ2)GO TO 30
15  DEL=DEL/2
  NCT=NCT+1
  IF(NCT.LE.LIM1)GO TO 2
  PRINT*,7HERROR 1,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
  RETURN
30  DO 35 I=1,IP
  DDD(I)=ABS(D(I)/AA(I))
35  IF(DDD(I).GT.EE)GO TO 20
  GO TO 4
20  MCT=MCT+1
  IF(MCT.LE.LIM2)GO TO 9
  PRINT*,7HERROR 2,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
  RETURN
9  RJ2=RJ1
  NCT=J
  DEL=1.
DO 25 I=1,IP
25  AA(I)=A(I)
  GO TO 1
40  PRINT45,(A(I),I=1,IP),NCT,MCT
  PRINT*,3HAA=,AA
  PRINT*,3H D=,D
  PRINT*,5HJMIN=,RJ1
45  FORMAT(1X,3G12.5,2X,2I5)
  END
SUBROUTINE COEF(W,RJ)
C  COMPUTE MEAN TRACKING ERROR FOR GIVEN PARAMETERS
C  EXPRESS PARTIAL DERIVATIVES OF MEAN TRACKING ERROR WRT
C  KGAIN AND GAM1A1 IN MATRIX B

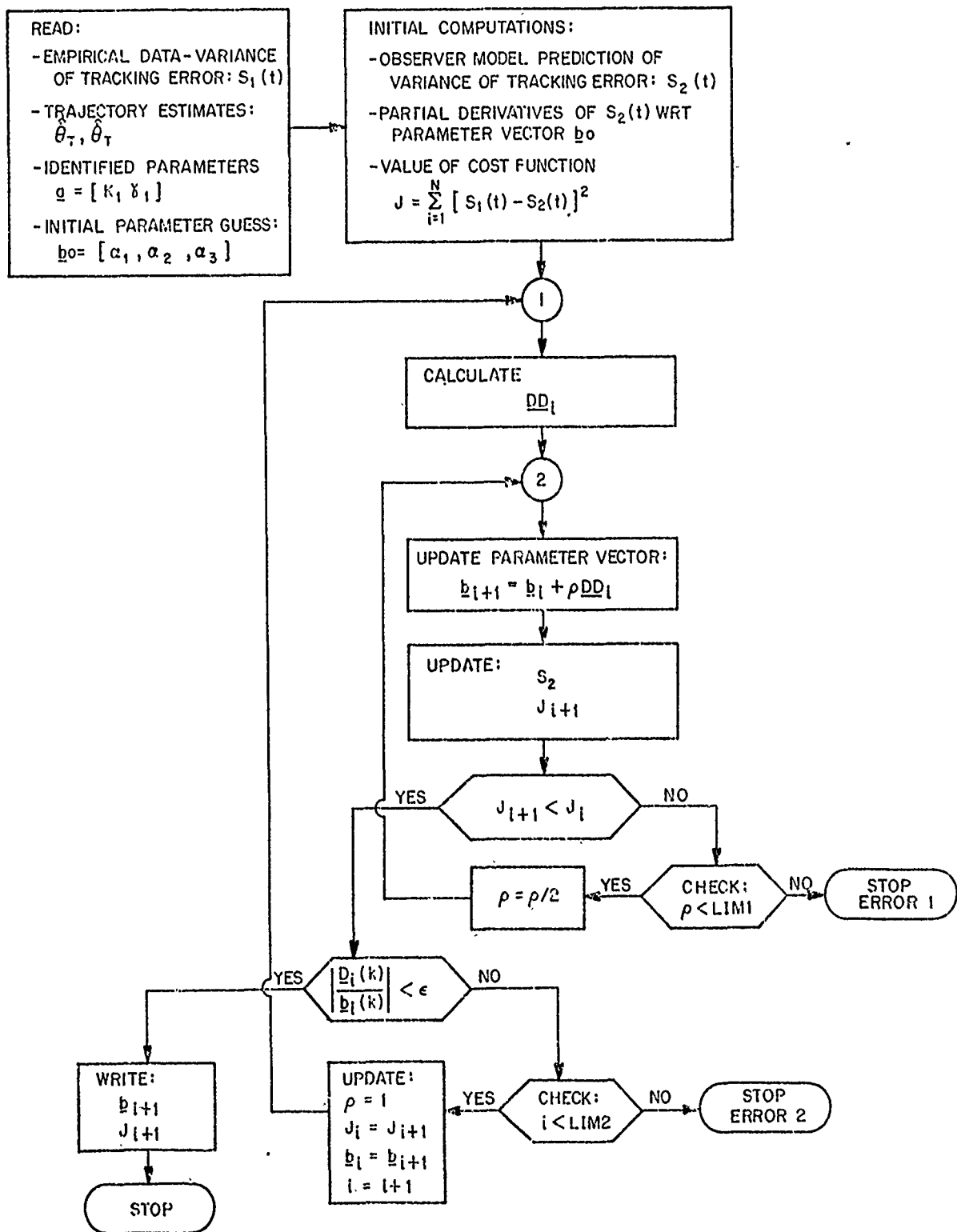
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DIMENSION W(2),IWK(12)
COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL,N1,N2
COMMON/MAT/F2(1,24),A(2),AA(2),B(1024,2),D(2),F3(4096)
U=W(1)
V=W(2)
X=U+V
REWIND 1
DO 10 I=1,N
WRITE(1)F3(I+N)
T=(I-1)*H
B(I)=F2(I)
S1=S2=1J0.
IF(U*T.GE.2J0.)S1=0.
IF(V*T.LE.-2J0.)S2=0.
IF(S1.NE.0.)S1=EXP(-U*T)
IF(S2.NE.0.)S2=EXP(V*T)
10 F3(I)=((X*T+1.)*S1-S2)/(X*X)
CALL VCONVJ(F3,9,N,N,IWK)
DO 13 I=1,N
FF=F3(I)*H
C H=TIME STEP .04
C INPUT TRAJECTORY INFORMATION AND EMPIRICAL DATA
WRITE(1)FF
B(I)=F2(I)
T=(I-1)*H
S1=S2=1J0.
IF(U*T.GE.2J0.)S1=0.
IF(V*T.LE.-2J0.)S2=0.
IF(S1.NE.0.)S1=EXP(-U*T)
IF(S2.NE.0.)S2=EXP(V*T)
15 F3(I)=((X*T-1.)*S2+S1)/(X*X)
CALL VCONVJ(F3,3,N,N,IWK)
DO 20 I=1,N
FF=F3(I)*H
WRITE(1)FF
B(I)=F2(I)
T=(I-1)*H
S1=S2=1J0.
IF(U*T.GE.2J0.)S1=0.
IF(V*T.LE.-2J0.)S2=0.
IF(S1.NE.0.)S1=EXP(-U*T)
IF(S2.NE.0.)S2=EXP(V*T)
20 F3(I)=(S2-S1)/X
CALL VCONVJ(F3,3,N,N,IWK)
REWIND 1
RJ=0.
DO 60 I=1,N
READ(1)F3(I+N)
60 CONTINUE
DO 70 I=N1,N2
F3(I)=F3(I)*H
70 RJ=RJ+(F3(I+N)-F3(I))**2
DO 65 I=1,2
DO 65 J=1,N
65 READ(1)3(J,I)
END

```

B. Flow Chart of Parameter Identification for Parameters Associated with Standard Deviation of Tracking Error Equations.



Comments: Execution of Program Fit 1.

1. Compile Program
2. Input: Tape 1 contains
  - a. Azimuth and elevation trajectory information
  - b. Empirical standard deviation of tracking error
3. Load Library Routines:
  - a. VCONVO - convolution integral computation from International Mathematical & Statistical Library (IMSL).
  - b. GMINV - matrix inverse routine from D. L. Kleinman Library [12].
4. Execute
5. Output: Parameter values for  $[\alpha_1, \alpha_2, \alpha_3]$ .

```

PROGRAM FIT(INPUT,OUTPUT,TAPE1)
C INPUT MEAN TRACKING ERROR PARAMETERS
PRINT*,30H TYPE IN GAMMA1 & KGAIN
READ*,A1,A2
REWIND 1
CALL CONVO(A1,A2)
1 PRINT*,50H TYPE 1 TO GO OR TYPE 0 TO STOP
READ*,IFG
IF(IFG.EQ.0) STOP
CALL INIT
GO TO 1
END
SUBROUTINE INIT
C INITIALIZATION ROUTINE
C READ IN DATA TO BE FITTED
C MAKE INITIAL GUESS
COMMON/VAR/LIM1,LIM2,EE,H,N,DEL,N1,N2
COMMON/MAT/A(3),AA(3),B(1024,4),D(3),F2(1024)
IP=3
PRINT*,30H TYPE IN LIM1,LIM2,EE
READ*,LIM1,LIM2,EE
PRINT*,3H N=,N,5HLIM1=,LIM1,5HLIM2=,LIM2,3HEE=,EE,3HIP=,IP
PRINT*,50H TYPE IN INITIAL GUESS--ALPHA1,ALPHA2,ALPHA3
READ*,AA
PRINT*,10H 1ST GUESS,AA
DEL=1
CALL COEF(AA,RJ2)
PRINT*,10H JMIN(1)= ,RJ2
CALL LOOP(RJ2,IP)
END
SUBROUTINE DD(IP)
C CALCULATE D MATRIX FROM Q AND R
DIMENSION R(3),Q(3,3),W1(3,3)
COMMON/VAR/LIM1,LIM2,EE,H,N,DEL,N1,N2
COMMON/MAT/A(3),AA(3),B(1024,4),D(3),F2(1024)
COMMON/MAIN1/NDIM,NDIM1
NDIM=IP,NDIM1=IP+1
DO 10 I=1,IP
R(I)=0.
DO 10 J=1,IP
10 Q(I,J)=0.
DO 35 K=N1,N2
SN=B(K,4)-E2(K)
DO 30 I=1,IP
DO 25 J=1,IP
25 Q(I,J)=Q(I,J)+B(K,I)*B(K,J)
30 R(I)=R(I)+B(K,I)*SN
35 CONTINUE
CALL GMINV(IP,IP,Q,W1,MR,0)
IP1=IP*IP
DO 50 I=1,IP
II=1

```

```

D(I)=0.
DO 45 J=1,IP1,IP
D(I)=D(I)+W1(J)*R(II)
45 II=II+1
50 D(I)=-D(I)
END
SUBROUTINE LOOP(RJ2,IP)
C ITERATION PROCESS
C COMPUTE A(I+1)=A(I)+D(I)
C DETERMINE WHEN A(I+1) IS ACCEPTABLE
DIMENSION DDD(3)
COMMON/VAR/LIM1,LIM2,EE,H,N,DEL,N1,N2
COMMON/MAT/A(3),AA(3),B(1024,4),D(3),F2(1024)
NCT=MCT=0.
1 CALL DD(IP)
2 DO 100 I=1,IP
D(I)=D(I)*DEL
100 A(I)=AA(I)+D(I)
IF(A(1).LT.3..OR.A(2).LT.0..OR.A(3).LT.0.)GO TO 7
CALL COEF(A,RJ1)
6 IF(RJ1.LT.RJ2)GO TO 30
7 DEL=DEL/2
NCT=NCT+1
IF(NCT.LE.LIM1)GO TO 2
PRINT*,7HERROR 1,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
RETURN
30 DO 35 I=1,IP
DDD(I)=ABS(D(I)/AA(I))
35 IF(DDD(I).GT.EE)GO TO 20
GO TO 40
20 MCT=MCT+1
IF(MCT.LE.LIM2)GO TO 9
PRINT*,7HERROR 2,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
RETURN
9 RJ2=RJ1
NCT=0
DEL=1.
DO 25 I=1,IP
25 AA(I)=A(I)
GO TO 1
40 PRINT*,4H A=,A,4HNCT=,NCT,4HMCT=,MCT
PRINT*,3HAA=,AA
PRINT*,3H D=,D
PRINT*,5HJMIN=,RJ1
END
SUBROUTINE COEF(W,SUM)
C COMPUTE TRACKING ERROR VARIANCE EQUATION
C EVALUATE COST FUNCTION
DIMENSION W(3)
COMMON/VAR/LIM1,LIM2,EE,H,N,DEL,N1,N2
COMMON/MAT/A(3),AA(3),B(1024,4),D(3),F2(1024)

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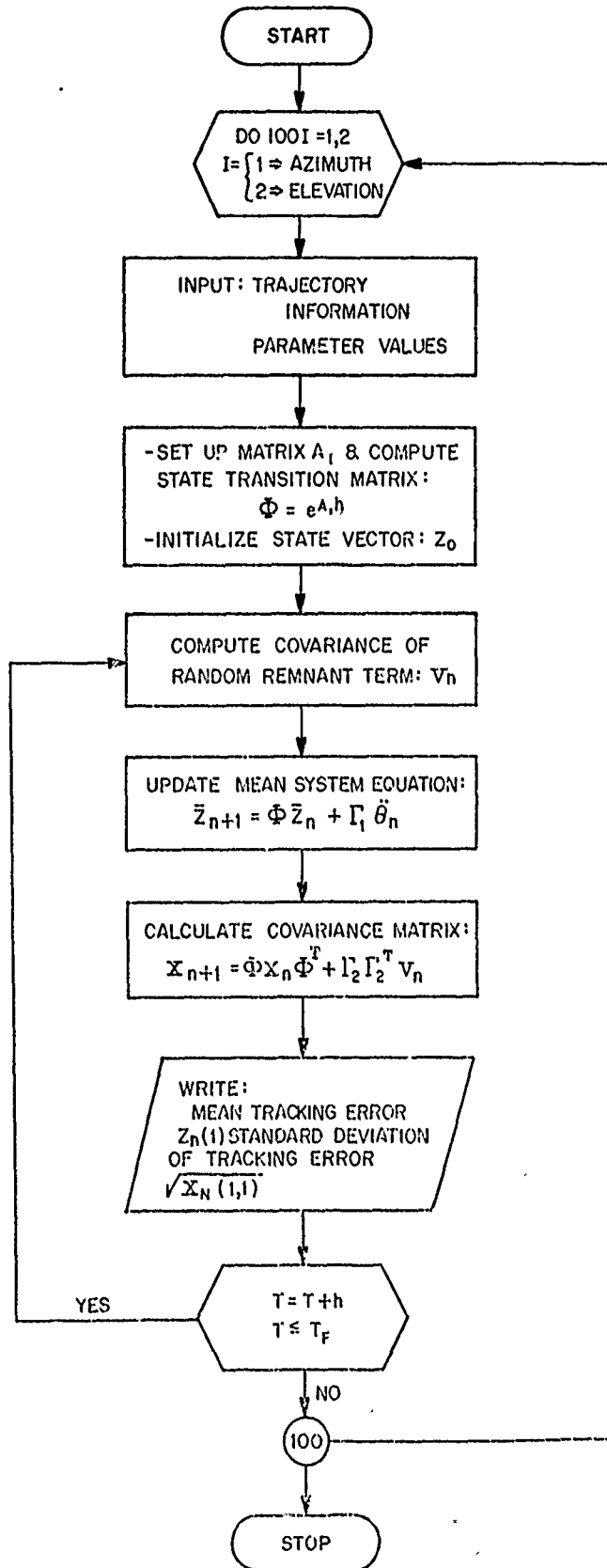
SUM=0.
DO 10 I=N1,N2
  B(I,4)=B(I,1)*W(1)+B(I,2)*W(2)+B(I,3)*W(3)
  SUM=SUM+(B(I,4)-F2(I))**2
10 CONTINUE
END
SUBROUTINE CONVO(A1,A2)
C  COMPUTE PARTIAL DERIVATIVES OF VARIANCE OF TRACKING ERROR WRT ALPHA1,2,3
C  DERIVATIVES ARE CONSTANT SINCE GAMMA1 AND KGAIN ARE KNOWN
C  INPUT TRAJECTORY INFORMATION AND EMPIRICAL DATA
  DIMENSION IWK(12),F3(2048)
  COMMON/VAR/LIM1,LIM2,EE,H,N,DEL,N1,N2
  COMMON/MAT/A(3),AA(3),B(1024,4),D(3),F2(1024)
  DATA N,H,N1,N2/1024,.04,100,1024/
  DO 15 I=1,N
    READ(1,*)F3(I),F2(I),Z,Z1
    T=(I-1)*H
    B(I)=((A1*EXP(A1*T)+A2*EXP(-A2*T))/(A1+A2))**2
15  CONTINUE
    CALL VCONVO(B,F3,N,N,IWK)
    DO 35 I=1,N
      F3(I)=F2(I)
35  READ(1,*)F2(I),Z
      REWIND 1
      DO 20 I=1,N
        BB=B(I)*H
        WRITE(1)BB
        I=(I-1)*H
20  B(I)=((A1*EXP(A1*T)+A2*EXP(-A2*T))/(A1+A2))**2
        CALL VCONVO(B,F3,N,N,IWK)
        DO 25 I=1,N
          T=(I-1)*H
          BB=B(I)*H
          WRITE(1)BB
          F3(I)=1.
25  B(I)=((A1*EXP(A1*T)+A2*EXP(-A2*T))/(A1+A2))**2
        CALL VCONVO(B,F3,N,N,IWK)
        REWIND 1
        DO 30 I=1,N
          F2(I)=F2(I)*F2(I)
30  B(I)=B(I)*H
        DO 40 J=2,3
          DO 40 I=1,N
40  READ(1)B(I,J)
      END

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# APPENDIX D

## Flow Chart and Program Listing of Computer Simulation of AAA Tracking Task



Comments: Execution of Program OBS.

1. Compile Program
2. Input: Tape 2 contains
  - a. Azimuth and elevation trajectory information
3. Load Library Routine:
  - a. DSCRT - computation of transition matrix from D. L. Kleinman Library [12].
4. Execute
5. Output: Time, mean tracking error, standard deviation for azimuth and elevation.

PROGRAM OBS(INPUT,OUTPUT,TAPE2)

DIMENSION A(3,3),Z(3),X(3,3),R1(3),W1(3,3),W2(3,3),P(7,2)

COMMON/MAIN1/N,N2

DATA P/-2.87,-1.,2.94,0.,.0496,.0024,.103,-3.81,-1.,3.02,0.,.0033  
1 .00047,.259/,DEL,N,TEND/,04,3,45./

C REDUCED ORDER MODEL SIMULATION FOR S60 SYSTEM

C P(1,IC)=GAMMA1 P(2,IC)=GAMMA2 P(3,IC)=KGAIN

C P(5,IC)=ALPHA1 P(6,IC)=ALPHA2 P(7,IC)=ALPHA3

C MODEL USES N=3 STATES AND DEL=.04 TIME STEP

N1=N\*N 3 N2=N+1

L1=1 3 L2=2

PRINT\*,50H TYPE 1 FOR AZ 2 FOR EL 3 FOR BOTH

READ\*,IFG

IF(IFG.EQ.1)L2=1

IF(IFG.EQ.2)L1=2

DO 500 IC=L1,L2

REWIND 2

Z(1)=-1.

Z(2)=Z(3)=P(3,IC)

DO 10 J=1,N1

10 A(J)=X(J)=0.

A(1)=P(1,IC)

A(7)=1.

A(2)=-P(1,IC)\*P(3,IC)

A(8)=A(9)=-P(3,IC)

C COMPUTE TRANSITION MATRIX - W1

CALL DSCRT(N,A,DEL,W1,W2,10)

C CALCULATE CONSTANT MATRICES

DO 50 I=1,N

II=1

R1(II)=0.

DO 45 J=I,N1,N

R1(II)=R1(II)+W2(J)\*Z(II)

45 II=II+1

50 CONTINUE

DO 60 I=1,N

Z(I)=0.

DO 60 J=1,N

60 A(I,J)=R1(I)\*R1(J)

DO 20 I=1,N

I1=I+N

I2=I1+N

20 R1(I1)=W2(I1)+W2(I2)

I=0.

C INPUT TRAJECTORY INFORMATION AND CALCULATE NOISE COVARIANCE

1 READ(2,3)C1,C2,P(4,1),C3,C4,P(4,2)

3 FORMAT(6G12,4)

P4=Z(2)-Z(3)+P(3,IC)\*Z(1)

P5=(P4-PP4)/DEL

V=(P(5,IC)+P(6,IC)\*P4\*P4+P(7,IC)\*P5\*P5)/DEL

PP4=P4

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DO 25 I=1,N
  II=1
  W2(I)=0.
  DO 15 J=I,N1,N
    W2(I)=W2(I)+W1(J)*Z(II)
  15  II=II+1
  25  CONTINUE
  C    UPDATE MEAN ERROR EQUATION
  DO 35 I=1,N
    Z(I)=W2(I)+R1(I)*P(4,IC)
  35  CONTINUE
  C    UPDATE ERROR COVARIANCE EQUATION
  CALL MULT(W1,X,N,N1,W2)
  DO 40 I=1,N1
    40  X(I)=A(I)*V+W2(I)
    SD=SQRT(X(1,1))
  C    OUTPUT MEAN ERROR TRACKING ERROR-Z(1), AND STANDARD DEVIATION OF
  C    TRACKING ERROR=SQUARE ROOT OF X(1,1)
  T=T+DEL
  LK=(T+.001)/DEL
  IF(MOD(LK,25).EQ.0)PRINT75,T,Z(1),SD
  75  FORMAT(5X,3G12.4)
  IF(T.GE.TEND)GO TO 500
  GO TO 1
  500 CONTINUE
  END
  SUBROUTINE MULT(E,F,L,L1,H)
  DIMENSION E(L1),F(L1),G(9),H(L1)
  C    MATRIX MANIPULATION  H=EFE'
  DO 10 I=1,L
    II=1
    DO 10 K=1,L
      TEMP=0.
      DO 5 J=I,L1,L
        TEMP=TEMP+E(J)*F(II)
      5  II=II+1
      KK=(K-1)*L+I
      10  G(KK)=TEMP
      DO 20 I=1,L
        DO 20 K=I,L
          TEMP=0.
          II=K
          DO 15 J=I,L1,L
            TEMP=TEMP+G(J)*E(II)
          15  II=II+L
          KK=(K-1)*L+I
          20  H(KK)=TEMP
          L2=L-1
          DO 30 I=1,L2
            L3=I+1
            DO 30 J=L3,L
              K1=(I-1)*L+J
              K2=(J-1)*L+I
              30  H(K1)=H(K2)
          END

```

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